Supplementary Material: Online Appendix

– NOT INTENDED FOR PUBLICATION –

Estimating the Gains from Trade in Frictional Local Labor Markets

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Contents

A Theoretical Framework

This section contains details of the model and of derivations that were omitted in the main text. The presentation is not necessarily self-contained but rather complementary with Section 2 of the paper.

The demand structure, introduced in subsection A.1, is common to all the market structures considered in the paper. Subsections A.2 to A.9 focus on the case of monopolistic competition with free entry and heterogeneous firms (MC-FE-HET). Sections A.10 and A.11 consider the special cases of homogeneous firms (MC-FE-HOM) and restricted entry (MC-RE-HET), respectively. Finally, Section A.12 analyzes a perfectly competitive multi-industry Armington model with frictional labor markets (PC).

A.1 Demand

The preferences of the normative representative consumer in location n are described by a time-separable and stationary two-tier utility function

$$U_n = \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho}\right)^t \prod_{i=1}^{I} (Y_{int})^{\alpha_i}, \quad \sum_{i=1}^{I} \alpha_i = 1, \tag{A.1}$$

where the consumption of good i in period t is a CES aggregate

$$Y_{int} = \left[\int_{\omega \in \Omega_{int}} q_{int}(\omega)^{\frac{\sigma_i - 1}{\sigma_i}} d\omega\right]^{\frac{\sigma_i}{\sigma_i - 1}}, \quad \sigma_i > 1.$$

$q_{int}(\omega)$ denotes the consumption of variety ω of good i and Ω_{int} is the set of varieties available to the consumer. The latter is endogenous under MC-FE-HET and MC-FE-HOM, and exogenous under PC and MC-RE. The price index dual to Y_{int} is

$$P_{int} = \left[\int_{\omega \in \Omega_{int}} p_{int}(\omega)^{1-\sigma_i} d\omega\right]^{\frac{1}{1-\sigma_i}},$$

where $p_{int}(\omega)$ denotes the price of variety ω.

In each location, there is a sequence of markets in one-period-ahead claims to consumption of each good i. We assume that these assets are not tradable across locations. Let a_{int+1} denote the claims to time $t+1$ consumption of good i and Q_{int} denote the price of 1 unit of this asset at time t. Note that both quantity and price of this asset are state-independent in the absence of aggregate uncertainty, a property that holds in equilibrium. The consumer then faces a sequence of budget constraints

$$\sum_i P_{int} Y_{int} + a_{int+1} Q_{int} \leq \sum_i a_{int} P_{int} + W_{nt}, \quad t \geq 1,$$

where W_{nt} denotes aggregate income (labor income and aggregate profits, if any) in location n. We rule out Ponzi schemes by implicitly imposing a natural debt limit.

The first-order conditions with respect to Y_{mnt} for good $m \in \{1, ..., I\}$, and the Langrange multiplier η_{nt} for the time t budget constraint, can be expressed as

$$\alpha_m \left(\frac{1}{1+\rho}\right)^t \prod_{i=1}^{I} (Y_{int})^{\alpha_i} (Y_{mnt})^{-1} = \eta_{nt} P_{mnt}, \tag{A.2}$$

$$\sum_i P_{int} Y_{int} + a_{int+1} Q_{int} = \sum_i a_{int} P_{int} + W_{nt}. \tag{A.3}$$

In a stationary equilibruim, $a_{int+1} = 0$ for all i and t, and $W_{nt} = W_n$ for all t. Imposing these conditions in (A.3) and using (A.2) yields

$$\sum_{m=1}^{I} \alpha_m \left(\frac{1}{1+\rho}\right)^t \prod_{i=1}^{I} (Y_{int})^{\alpha_i} (\eta_{nt})^{-1} = W_n. \tag{A.4}$$

Let $\tilde{V}_{nt} = \prod_{i=1}^{I}(Y_{int})^{\alpha_i}$. Under stationarity, $\tilde{V}_{nt} = \tilde{V}_n$ and $Y_{mnt} = Y_{mn}$ for all t. Equation (A.4) then becomes

$$\left(\frac{1}{1+\rho}\right)^t = \frac{W_n}{\tilde{V}_n}\eta_{nt}. \tag{A.5}$$

Plugging (A.5) into (A.2) with $Y_{mnt} = Y_{mn}$ for all t, we obtain

$$\tilde{V}_n = \prod_{i=1}^{I} (\alpha_i)^{\alpha_i} \frac{W_n}{\prod_{i=1}^{I}(P_{in})^{\alpha_i}}. \tag{A.6}$$

In turn, plugging (A.6) into (A.1) yields V_n, the indirect utility function in the stationary equilibrium,

$$V_n = (\rho)^{-1} \prod_{i=1}^{I} (\alpha_i)^{\alpha_i} \frac{W_n}{\prod_{i=1}^{I}(P_{in})^{\alpha_i}}. \tag{A.7}$$

A.2 The Firm's Problem

Throughout this section, we consider a firm with poductivity φ in industry i located in city c.

A.2.1 The (Conditional) Revenue Function

Suppose that the firm is employing l production workers and serving a given set of markets at some point in time. Let $I_{icn}(\varphi)$ denote an export decision indicator for an arbitrary destination n. In this section, we take l and $I_{icn}(\varphi)$ as given and characterize the optimal allocation of workers across destinations served by the firm. This will allow us to derive the firm's revenue function conditional on l and $I_{icn}(\varphi)$.

Let $l_{icn}(\varphi)$ denote the mass of production workers allocated by the firm to serve market n. Then $l = \sum_n I_{icn}(\varphi) l_{icn}(\varphi)$. At a given point in time, the firm's revenue, output and demand in any destination n can be written, respectively, as

$$r_{icn}(\varphi) \equiv p_{icn}(\varphi) q_{icn}(\varphi), \tag{A.8}$$

$$y_{icn}(\varphi) = l_{icn}(\varphi)\varphi, \tag{A.9}$$

$$q_{icn}(\varphi) = X_{in} \frac{(p_{icn}(\varphi))^{-\sigma_i}}{(P_{in})^{1-\sigma_i}}, \tag{A.10}$$

where $l_{icn}(\varphi)$ is the mass of production workers hired by the firm to serve market n.[1] Moreover, due to transportation costs,

$$q_{icn}(\varphi) = y_{icn}(\varphi)(\tau_{icn})^{-1}. \tag{A.11}$$

[1] Note that these expressions apply at any given point in time t, not just in stationary equilibrium. Because in this section we focus on a static problem, however, we simplify notation by omitting the time index.

Using (A.9), (A.10) and (A.11)

$$q_{icn}(\varphi) = l_{icn}(\varphi)\varphi(\tau_{icn})^{-1}, \tag{A.12}$$

$$p_{icn}(\varphi) = \left(\frac{l_{icn}(\varphi)\varphi}{\tau_{icn}A_{in}}\right)^{-\frac{1}{\sigma_i}}, \tag{A.13}$$

where

$$A_{in} = X_{in}\left(P_{in}\right)^{\sigma_i-1}, \tag{A.14}$$

is the industry-specific demand shifter in destination n.

Equations (A.12) and (A.13), imply that revenue from sales in n can be written as a function of $l_{icn}(\varphi)$,

$$r_{icn}(\varphi) = (A_{in})^{\frac{1}{\sigma_i}}\left(\frac{l_{icn}(\varphi)\varphi}{\tau_{icn}}\right)^{\frac{\sigma_i-1}{\sigma_i}}. \tag{A.15}$$

Using (A.13), we can then express the marginal revenue of allocating an additional production worker to serve market n as

$$\frac{\partial r_{icn}(\varphi)}{\partial l_{icn}(\varphi)} = p_{icn}(\varphi)\left(\frac{\varphi}{\tau_{icn}}\right)\left(\frac{\sigma_i-1}{\sigma_i}\right).$$

An efficient allocation of workers requires equating marginal revenue across all destinations. This implies

$$p_{icn}(\varphi) = \tau_{icn}p_{icc}(\varphi), \tag{A.16}$$

for all n. Using (A.13) and (A.16), relative employment across any two destinations n and $n^{'}$ served by the firm can be written as

$$\frac{l_{icn}(\varphi)}{l_{icn'}(\varphi)} = \frac{A_{icn}}{A_{icn'}}\left(\frac{\tau_{icn}}{\tau_{icn'}}\right)^{1-\sigma_i}.$$

For $n^{'} = c$, $\tau_{icc} = 1$ implies

$$l_{icn}(\varphi) = l_{icc}(\varphi)\left(\tau_{icn}\right)^{1-\sigma_i}\left(\frac{A_{in}}{A_{ic}}\right). \tag{A.17}$$

Using $l = \sum_n I_{icn}(\varphi)l_{icn}(\varphi)$,

$$l_{icc}(\varphi) = \frac{A_{ic}}{\sum_{n'} I_{icn'}(\varphi)A_{in'}\left(\tau_{icn'}\right)^{1-\sigma_i}}l. \tag{A.18}$$

Moreover, substituting (A.18) into (A.17) yields

$$l_{icn}(\varphi) = \frac{\left(\tau_{icn}\right)^{1-\sigma_i}A_{in}}{\sum_{n'} I_{icn'}(\varphi)A_{in'}\left(\tau_{icn'}\right)^{1-\sigma_i}}l. \tag{A.19}$$

The firm's total revenue conditional on l is $r_{ic}(l;\varphi) = \sum_n r_{icn}(\varphi)I_{icn}(\varphi)$. Using (A.15) and (A.19), we can express it as

$$r_{ic}(l;\varphi) = \left[\sum_n I_{icn}(\varphi)A_{in}\left(\tau_{icn}\right)^{1-\sigma_i}\right]^{\frac{1}{\sigma_i}}(l\varphi)^{\frac{\sigma_i-1}{\sigma_i}}. \tag{A.20}$$

A.2.2 Optimal Vacancy Posting

We now study the dynamic behavior of the firm, taking all export decisions as given and constant over time. In a stationary equilibrium, the firm faces a time-invariant revenue function given by (A.20). The firm determines its optimal employment of production workers by posting vacancies, denoted v, with the goal of maximizing the present value of expected profits. We show that a firm that starts with no production workers reaches its optimal long-run level in the following period.

Suppose that the firm is currently employing l production workers. Then it solves

$$\Pi_{ic}(l;\varphi) = \max_{v} \frac{1}{1+\rho}\left\{ r_{ic}(l;\varphi) - w_{ic}(l;\varphi)l - p_{ic}^{M}\sum_{n} I_{icn}(\varphi) f_{icn} - p_{ic}^{V} v + (1-\delta_c)\,\Pi_{ic}(l';\varphi)\right\},$$
$$s.t. \quad l' = l + m_c(\theta_c)v,$$

where $l^{'}$ is the level of employment next period.

The first order condition for vacancy posting can be written as:

$$(1-\delta_c)\,\frac{\partial \Pi_{ic}(l';\varphi)}{\partial l'} = \frac{p_{ic}^{V}}{m_c(\theta_c)}, \tag{A.21}$$

Note that optimal employment size is independent of current employment l and constant over time as long as the firm is not forced to exit the market. Hence the firm converges to its optimal employment size in one period. From this point on, $l = l^{'}$. Using this condition and the envelope theorem yields

$$\frac{\partial \Pi_{ic}(l;\varphi)}{\partial l} = \frac{1}{\rho+\delta_c}\left[\frac{\partial r_{ic}(l;\varphi)}{\partial l} - w_{ic}(l;\varphi) - \frac{\partial w_{ic}(l;\varphi)}{\partial l} l\right]. \tag{A.22}$$

Combining (A.21) and (A.22) with $l = l^{'}$, we can obtain the implicit optimal pricing rule of the firm,

$$\frac{\partial r_{ic}(l;\varphi)}{\partial l} = \frac{\partial w_{ic}(l;\varphi)}{\partial l} l + w_{ic}(l;\varphi) + \frac{p_{ic}^{V}}{m_c(\theta_c)}\left(\frac{\rho+\delta_c}{1-\delta_c}\right). \tag{A.23}$$

A.2.3 Bargaining

This section follows the analysis in Stole & Zwiebel (1996) and Felbermayr et al. (2011).[2] We assume that the bargaining outcome over the division of the total surplus from a match satisfies the following surplus-splitting rule:

$$(1-\beta_i)\left[E_{ic}(l;\varphi) - U_c\right] = \beta_i \frac{\partial \Pi_{ic}(l;\varphi)}{\partial l}, \tag{A.24}$$

where U_c is the worker's outside option (i.e. the value of unemployment) and $E_{ic}(l;\varphi)$ is the value of employment in a firm with productivity φ and l production workers. The Bellman equation for workers can be written as:

$$E_{ic}(l;\varphi) - U_c = \frac{w_{ic}(l;\varphi) - t_c - \rho U_c}{(\rho+\delta_c)}. \tag{A.25}$$

Inserting (A.22) and (A.25) into (A.24) yields

$$w_{ic}(l;\varphi) = (1-\beta_i)\,(t_c + \rho U_c) + \beta_i \frac{\partial r_{ic}(l;\varphi)}{\partial l} - \beta_i \frac{\partial w_{ic}(l;\varphi)}{\partial l} l. \tag{A.26}$$

[2] Wage agreements can be renegotiated any time before production begins. A firm may fire an employee or the latter may quit, in which case the worker immediately returns to the unemployment pool. During the bargaining process, the firm cannot recruit additional workers. Once production begins, wage agreements become binding. In equilibrium, wages are immune to intra-firm pairwise renegotiations.

Using the revenue function (A.20), one can verify by direct substitution that

$$w_{ic}(l;\varphi) = (1-\beta_i)(t_c+\rho U_c) + \left(\frac{\sigma_i}{\sigma_i-\beta_i}\right)\frac{\partial r_{ic}(l;\varphi)}{\partial l}\beta_i, \tag{A.27}$$

solves (A.26). Differentiating this equation with respect to l, we obtain:

$$\frac{\partial w_{ic}(l;\varphi)}{\partial l}l = -\left(\frac{\beta_i}{\sigma_i-\beta_i}\right)\frac{\partial r_{ic}(l,\varphi)}{\partial l}.$$

Substituting this expression in (A.23) yields

$$w_{ic}(l;\varphi) = \left(\frac{\sigma_i}{\sigma_i-\beta_i}\right)\frac{\partial r_{ic}(l;\varphi)}{\partial l} - \frac{p_{ic}^V}{m_c(\theta_c)}\left(\frac{\rho+\delta_c}{1-\delta_c}\right). \tag{A.28}$$

From (A.27) and (A.28), it follows that the equilibrium wage does not vary across firms within city-industry cells. We can then express the Wage Curve as a function of θ:

$$w_{ic} = t_c + \rho U_c + \frac{\beta_i}{(1-\beta_i)}\left(\frac{\rho+\delta_c}{1-\delta_c}\right)\frac{p_{ic}^V}{m_c(\theta_c)}. \tag{A.29}$$

Finally, for future purposes, we express industry-city wages w_{ic} as a function of the city's employment rate, denoted $e_c \equiv \sum_i L_{ic}/L_c$, and aggregate expenditure, denoted X_c. To do this, we first write the Bellman equation for unemployed workers, imposing wage equalization within industry-city cells:

$$\rho U_c = b_c + \frac{\theta_c m_c(\theta_c)}{\rho+\delta_c}\sum_i \eta_{ic}(w_{ic}-t_c-\rho U_c), \tag{A.30}$$

where $\eta_{ic} \equiv L_{ic}/\sum_i L_{ic}$ denotes the employment share of industry i and L_{ic} is the measure of workers employed in i. Next, we rewrite the city's expected gross wage for employed workers as

$$\sum_i \eta_{ic}w_{ic} = \sum_i L_{ic}w_{ic}/e_cL_c = X_c/e_cL_c,$$

where the second equality follows from the trade balance condition (A.55) in city c -see section section A.6. Finally, we impose fiscal balance, i.e. $(1-e_c)b_c = e_ct_c$, and make use of the Beveridge curve equation $(1-e_c)\theta_c m_c(\theta_c) = \delta_c e_c$. Using these results together with (A.30) to solve out the outside option U_c and tax t_c from (A.29) yields

$$w_{ic} = \left[\frac{\delta_c+\theta_c m_c(\theta_c)}{\rho+\delta_c+\theta_c m_c(\theta_c)}\right]\left[\frac{(\rho+\delta_c)b_c}{\theta_c m_c(\theta_c)}+\frac{X_c}{L_c}\right] + \frac{\beta_i}{(1-\beta_i)}\left(\frac{\rho+\delta_c}{1-\delta_c}\right)\frac{p_{ic}^V}{m_c(\theta_c)}. \tag{A.31}$$

A.2.4 Recruitment and Marketing Services

We now characterize the equilibrium prices of recruitment and marketing services in industry i of city c in terms of the equilibrium wage w_{ic} and tightness θ_c in the local labor market.

Recruitment. In any period along a stationary equilibrium path, a worker matched to industry i that self-selects into recruitment produces χ_{ic} vacancies and sells them at a price p_{ic}^V. The worker remains in this occupation with probability $1-\delta_c$ in subsequent periods. Under perfect competition and constant returns, a static zero-profit condition prevails in every period, hence

$$p_{ic}^V = \frac{w_{ic}}{\chi_{ic}},$$

where w_{ic} is the equilibrium wage common to all three possible occupational choices.

In the welfare analysis, we impose additional structure on the recruitment worker's productivity, setting

$$\chi_{ic}^{-1} = k_{ic} m_c(\theta_c), \tag{A.32}$$

where $k_{ic} > 0$ is a parameter. The assumption yields a recruitment cost per worker proportional to the industry-city wage; that is,

$$\frac{p_{ic}^V}{m_c(\theta_c)} = k_{ic} w_{ic}. \tag{A.33}$$

Marketing. An agency supplying marketing services to industry i can recruit a worker to produce one unit of the entry or fixed costs of firms in the industry. Letting $\Pi_{ic}^{M,f}$ denote the value of a filled vacancy in a stationary equilibrium,

$$\Pi_{ic}^{M,f} = \frac{1}{1+\rho}\left[p_{ic}^M - w_{ic} + (1-\delta_c)\Pi_{ic}^{M,f}\right]. \tag{A.34}$$

Under perfect competition and constant returns, the value of recruiting a marketing worker is equal to the recruitment cost. Since workers are hired one period before production takes place, the zero-profit condition for marketing services is

$$(1-\delta_c)\Pi_{ic}^{M,f} = \frac{p_{ic}^V}{m_c(\theta_c)}. \tag{A.35}$$

Solving for $\Pi_{ic}^{M,f}$ from (A.34) and substituting in (A.35) yields the equilibrium price of marketing services in cell ic,

$$p_{ic}^M = w_{ic} + \left(\frac{\rho+\delta_c}{1-\delta_c}\right)\frac{p_{ic}^V}{m_c(\theta_c)}. \tag{A.36}$$

A.2.5 Firm-level Outcomes

Upon entry, the firm starts with zero production workers but immediately recruits to achieve its optimal size, denoted $l_{ic}(\varphi)$, in the following period. Given a set of export decisions $I_{icn}(\varphi)$ for all n, the expected profits of the firm upon entry can be written as:

$$\Pi_{ic}(0;\varphi) = \frac{1}{1+\rho}\left[-\frac{p_{ic}^V}{m_c(\theta_c)} l_{ic}(\vartheta) + (1-\delta_c)\Pi_{ic}(\varphi)\right], \tag{A.37}$$

where

$$\Pi_{ic}(\varphi) = \frac{1}{1+\rho}\left[r_{ic}(\varphi) - w_{ic} l_{ic}(\varphi) - p_{ic}^M \sum_n I_{icn}(\varphi) f_{icn} + (1-\delta_c)\Pi_{ic}(\varphi)\right] \tag{A.38}$$

is the value function of the vacancy posting problem evaluated at the firm's (constant) optimal employment size. That is, $\Pi_{ic}(\varphi) = \Pi_{ic}(l_{ic}(\varphi);\varphi)$ and $r_{ic}(\varphi) = r_{ic}(l_{ic}(\varphi);\varphi)$, after a slight abuse of notation. Note that no recruitment costs are paid after the entry period since there are no match-specific separation shocks. Worker-firm matches survive until the firm is forced to exit (with per-period probability δ_c).

We can now define the cost of labor in industry i of city c, denoted μ_{ic}, as

$$\mu_{ic} \equiv w_{ic} + \left(\frac{\rho+\delta_c}{1-\delta_c}\right)\frac{p_{ic}^V}{m_c(\theta_c)}. \tag{A.39}$$

μ_{ic} can be interpreted as the per-period cost of hiring (recruitment plus employment wage) an additional production worker or marketing worker in industry i of city c. To see this, we use (A.36), (A.38) and (A.39) to rewrite (A.37) and obtain

$$\Pi_{ic}(0;\varphi) = \frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)}\pi_{ic}(\varphi), \tag{A.40}$$

where $\pi_{ic}(\varphi)$ is the stationary per-period profit of the firm (gross of the entry cost), defined by

$$\pi_{ic}(\varphi) = r_{ic}(\varphi) - \mu_{ic} l_{ic}(\varphi) - \mu_{ic} \sum_n I_{icn}(\varphi) f_{icn}. \tag{A.41}$$

Note that (A.39), (A.28) and the revenue equation (A.20) imply

$$\mu_{ic} = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right) \frac{r_{ic}(\varphi)}{l_{ic}(\varphi)}. \tag{A.42}$$

Substituting this into (A.41), we can rewrite the per-period profit function as in the main text,

$$\pi_{ic}(\varphi) = \left(\frac{1 - \beta_i}{\sigma_i - \beta_i}\right) r_{ic}(\varphi) - \mu_{ic} \sum_n I_{icn}(\varphi) f_{icn}. \tag{A.43}$$

Equations (A.20) and (A.42) allow us to compute the firm's optimal employment of production workers in terms of μ_{ic}

$$l_{ic}(\varphi) = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i} \frac{(\varphi)^{\sigma_i - 1}}{(\mu_{ic})^{\sigma_i}} \sum_n I_{icn}(\varphi) A_{in} \left(\tau_{icn}\right)^{1-\sigma_i}. \tag{A.44}$$

Using (A.42) and (A.44), yields the firm's per-period revenue

$$r_{ic}(\varphi) = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i - 1} \left(\frac{\varphi}{\mu_{ic}}\right)^{\sigma_i - 1} \sum_n I_{icn}(\varphi) A_{in} \left(\tau_{icn}\right)^{1-\sigma_i}. \tag{A.45}$$

Next, use (A.8) and (A.12) to obtain

$$p_{icn}(\varphi) = \frac{r_{icn}(\varphi)}{l_{icn}(\varphi)} \frac{\tau_{icn}}{\varphi}.$$

Combining this with (A.42) yields the profit maximizing price in terms of μ_{ic}

$$p_{icn}(\varphi) = \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1}\right) \left(\frac{\mu_{ic}}{\varphi}\right) \tau_{icn}. \tag{A.46}$$

Productivity Cutoffs and Export Decisions. The per-period profit function (A.43) and the revenue function (A.45) determine the productivity cutoffs, denoted φ^*_{icn}, such that a firm with productivity φ enters market n if and only if $\varphi \geq \varphi^*_{icn}$. That is,

$$\Lambda^0_i A_{in} \left(\tau_{icn}\right)^{1-\sigma_i} \left(\varphi^*_{icn}\right)^{\sigma_i - 1} \left(\mu_{ic}\right)^{-\sigma_i} = f_{icn}, \tag{A.47}$$

where $\Lambda^0_i = (1 - \beta_i)\left(\sigma_i - 1\right)^{\sigma_i - 1} / \left(\sigma_i - \beta_i\right)^{\sigma_i}$. The export decisions can then be written as

$$I_{icn}(\varphi) = \begin{cases} 1, & \text{if } \varphi \geq \varphi^*_{icn}, \\ 0, & \text{otherwise.} \end{cases} \tag{A.48}$$

A.3 Entry

A.3.1 The Cost of Entry

In order to discover and maintain its productivity over time, the firm commits to an investment of f^e_{ic} units of marketing services first incurred when the firm starts production (i.e. the period that immediately follows entry) and in each subsequent period in which the firm is active. This setting ensures that the per-period cost of entry is equal to the per-period cost of hiring production workers, a standard property in frictionless trade models.

The present value of the entry cost can be written as

$$\frac{1}{1+\rho}\left[\left(\frac{1-\delta_c}{1+\rho}\right)p_{ic}^M f_{ic}^e+\left(\frac{1-\delta_c}{1+\rho}\right)^2 p_{ic}^M f_{ic}^e+\left(\frac{1-\delta_c}{1+\rho}\right)^3 p_{ic}^M f_{ic}^e+......\right].$$

Equations (A.36) and (A.39) imply $p_{ic}^M=\mu_{ic}$, hence the present value of the entry cost can be written as a function of the local cost of labor, μ_{ic},

$$\frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)}f_{ic}^e\mu_{ic}.$$

A.3.2 The Free Entry Condition

Under free entry, the expected profits are equal to the present value of the entry cost. Using (A.40) and (A.41), the free entry condition can be written as

$$\frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)}\int_0^\infty \pi_{ic}(\varphi)dG_{ic}(\varphi)=\frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)}f_{ic}^e\mu_{ic},$$

where $\pi_{ic}(\varphi)$ is per-period profit. Substituting the revenue function (A.45) into the per-period profit function (A.43) and imposing $I_{icn}(\varphi)=1$ if $\varphi\geq\varphi_{icn}^*$, the free entry condition becomes

$$f_{ic}^e\mu_{ic}=\sum_n\int_{\varphi_{icn}^*}^\infty\left[\left(\frac{(\sigma_i-\beta_i)\tau_{icn}\mu_{ic}}{(\sigma_i-1)}\right)^{1-\sigma_i}A_{in}\,(\varphi)^{\sigma_i-1}\left(\frac{1-\beta_i}{\sigma_i-\beta_i}\right)-f_{icn}\mu_{ic}\right]dG_{ic}(\varphi).$$

Using the cutoff condition in destination n (equation (A.47) in the main text), we obtain

$$f_{ic}^e=\sum_n\int_{\varphi_{icn}^*}^\infty f_{icn}\left[\left(\frac{\varphi}{\varphi_{icn}^*}\right)^{\sigma_i-1}-1\right]dG_{ic}(\varphi).$$

Assume that $G_{ic}(\varphi)$ is a Pareto distribution, with shape parameter κ_i and lower bound $\varphi_{min,ic}$. If $\kappa_i>\sigma_i-1$, then the integral has a closed-form solution. In this case, the free entry condition simplifies to

$$f_{ic}^e=\frac{(\sigma_i-1)}{(\kappa_i-\sigma_i+1)}\sum_n f_{icn}\left(\frac{\varphi_{min,ic}}{\varphi_{icn}^*}\right)^{\kappa_i}. \tag{A.49}$$

A.4 Labor Demand and Supply

The stationary demand for production workers in industry i of city c can be computed as the sum of destination-specific labor demands for producers serving destination n:

$$\sum_n M_{ic}\frac{1-G_{ic}(\varphi_{icn}^*)}{1-G_{ic}(\varphi_{icc}^*)}\left[\int_{\varphi_{icn}^*}^\infty l_{icn}(\varphi)\frac{dG_{ic}(\varphi)}{1-G_{ic}(\varphi_{icn}^*)}\right], \tag{A.50}$$

where the demand for workers producing output for n in firm φ, $l_{icn}(\varphi)$, is obtained from (A.44) by setting $I_{icn}(\varphi)=1$ and $I_{icv}(\varphi)=0$ for $v\neq n$. M_{ic} is the mass of producers in cell ic. The term in brackets in (A.50) is then the average destination-specific demand for production workers across firms serving n.

Under Pareto productivity, we can evaluate the integral in (A.50) and obtain

$$M_{ic}\left(\frac{\sigma_i-1}{\sigma_i-\beta_i}\right)^{\sigma_i}\kappa_i\left[\sum_n\frac{X_{in}}{(P_{in})^{1-\sigma_i}}\frac{(\tau_{icn})^{1-\sigma_i}}{(\mu_{ic})^{\sigma_i}}(\varphi_{icc}^*)^{\kappa_i}\frac{(\varphi_{icn}^*)^{\sigma_i-\kappa_i-1}}{\kappa_i-\sigma_i+1}\right].$$

Using the export cutoff conditions yields a simplified expression for the demand for production workers

$$M_{ic}\frac{(\sigma_i-1)\kappa_i}{(\kappa_i-\sigma_i+1)(1-\beta_i)}\sum_n\left(\frac{\varphi^*_{icc}}{\varphi^*_{icn}}\right)^{\kappa_i}f_{icn}.$$

In turn, the industry's stationary labor demand due to fixed and entry marketing costs is:[3]

$$\frac{M_{ic}}{1-G_{ic}(\varphi^*_{icc})}f^e_{ic}+\sum_n M_{ic}\left[\frac{1-G_{ic}(\varphi^*_{icn})}{1-G_{ic}(\varphi^*_{icc})}\right]f_{icn}.$$

The final component of the aggregate demand for labor in cell ic is driven by recruitment services. Due to exogenous job destruction, a fraction δ_c of the stationary workforce employed as production and marketing workers needs to be replaced. Each replacement requires posting $\delta_c/m_c(\theta_c)$ vacancies in the local labor market. Producing a vacancy in turn requires the employment of χ_{ic}^{-1} recruitment workers. Under assumption (A.32), the industry's demand for recruitment workers is equal to a constant proportion $\delta_c k_{ic}$ of its demand for production and marketing workers.

We can now show that M^e_{ic}, the mass of entrants, is proportional to L_{ic}, the mass of workers that are matched to the industry. Given our derivations in this section, equating L_{ic} to the aggregate labor demand in industry i of city c yields

$$L_{ic}=(1+\delta_c k_{ic})M_{ic}\left[\sum_n(\frac{\varphi^*_{icc}}{\varphi^*_{icn}})^{\kappa_i}f_{icn}\left(\frac{(\sigma_i-1)\kappa_i}{(\kappa_i-\sigma_i+1)(1-\beta_i)}+1\right)+\frac{f^e_{ic}}{(\frac{\varphi_{min,ic}}{\varphi^*_{icc}})^{\kappa_i}}\right].$$

Imposing the free entry condition (A.49) and the aggregate stability condition

$$\delta_c M_{ic}=[1-G_{ic}(\varphi^*_{icc})]M^e_{ic}, \tag{A.51}$$

we obtain:

$$M^e_{ic}=\frac{\delta_c}{(1+\delta_c k_{ic})}\left[\frac{1}{1-\beta_i}+\frac{1}{\sigma_i-1}\right]^{-1}\frac{L_{ic}}{\kappa_i f^e_{ic}}. \tag{A.52}$$

A.5 Price Index

The price index in industry i of city c can be expressed as follows:

$$P_{in}^{1-\sigma_i}=\sum_v M_{iv}\left[\frac{1-G_{iv}(\varphi^*_{inv})}{1-G_{iv}(\varphi^*_{ivv})}\right]\int_{\varphi^*_{ivn}}^{\infty}p_{ivn}(\varphi)^{1-\sigma_i}dG_{iv}(\varphi|\varphi\geq\varphi^*_{inv}).$$

Substituting optimal firm prices (A.46) and imposing Pareto productivity yields:

$$P_{in}^{1-\sigma_i}=\frac{\kappa_i}{\kappa_i-\sigma_i+1}\left(\frac{\sigma_i-\beta_i}{\sigma_i-1}\right)^{1-\sigma_i}\sum_v M_{iv}(\varphi^*_{ivv})^{\kappa_i}(\varphi^*_{ivn})^{\sigma_i-\kappa_i-1}\left(\mu_{iv}\tau_{ivn}\right)^{1-\sigma_i}.$$

Using the export cutoff condition, the price index becomes

$$P_{in}^{-\kappa_i}=\frac{\kappa_i}{\kappa_i-\sigma_i+1}\left(\frac{\sigma_i-\beta_i}{\sigma_i-1}\right)^{-\kappa_i}\left(\frac{\sigma_i-\beta_i}{1-\beta_i}\right)^{1-\frac{\kappa_i}{\sigma_i-1}}\sum_v M_{iv}(\varphi^*_{ivv})^{\kappa_i}\left(\mu_{iv}\tau_{ivn}\right)^{-\kappa_i}\left(\frac{\mu_{iv}f_{ivn}}{X_{in}}\right)^{1-\frac{\kappa_i}{\sigma_i-1}}. \tag{A.53}$$

[3]It is straightforward to verify that, under the entry protocol described in section A.3.1, the industry's demand for entry workers is equal to $M^e_{ic}f^e_{ic}/\delta_c$ in the stationary equilibrium.

A.6 Closing the Model

In this section, we show how to close the MC-FE-HET model and compute all equilibrium variables. We exploit the block recursive structure of the model.

Step 1: Given labor market tightness θ_c and aggregate expenditure X_c in any c, we use (A.31) and (A.39) to solve for the wage w_{ic} and cost of labor μ_{ic} in every industry-city cell, each as a function of (θ_c, X_c). This requires using the city-specific Beveridge curve,

$$(1-e_c)\,\theta_c m_c(\theta_c) = \delta_c e_c, \tag{A.54}$$

to eliminate e_c from (A.31). Equation (A.54) is a stability condition that requires identical flows into and out of unemployment in a stationary equilibrium.

Step 2: Substituting solutions from step 1 into the cutoff (A.47) and free entry (A.49) conditions, we solve for all productivity cutoffs φ^*_{icn} and demand shifters A_{in}, each as a function of tightness and expenditure in all cities, i.e. $\{(\theta_c, X_c)\}_{c=1}^{C}$.

Step 3: Substituting solutions from steps 1-2 into (A.43)-(A.46) and (A.48), we solve for all firm-level outcomes: employment $l_{ic}(\varphi)$, revenue $r_{ic}(\varphi)$, profit $\pi_{ic}(\varphi)$, price $p_{ic}(\varphi)$ and export decisions $I_{icn}(\varphi)$, each as a function of $\{(\theta_c, X_c)\}_{c=1}^{C}$.

Step 4: Substituting solutions from steps 1-3 into (A.51)-(A.53) and (A.14), we solve for the price index P_{ic} and measures of producers M_{ic}, entrants M^e_{ic} and employment L_{ic} in all industry-city cells, each as a function of $\{(\theta_c, X_c)\}_{c=1}^{C}$. This step requires imposing the Cobb-Douglas expenditure shares $X_{ic} = \alpha_i X_c$ in (A.14) and (A.53).

Step 5: Substituting L_{ic} from step 4 and e_c from (A.54) into $e_c L_c = \sum_i L_{ic}$ (definition of employment rate), we solve for tightness θ_c in every city as a function of $\{X_c\}_{c=1}^{C}$. Using the latter, we solve out θ_c from all variables computed thus far in steps 1-4, expressing them solely as functions of $\{X_c\}_{c=1}^{C}$.

Step 6: Finally, substituting for wages and employment from step 5 into city-specific trade balance conditions,

$$X_c = \sum_i L_{ic} w_{ic}. \tag{A.55}$$

we solve for aggregate expenditure in all cities, $\{X_c\}_{c=1}^{C}$. The right-hand side of (A.55) follows from fiscal balance (unemployment benefits and labor taxes sum to zero), hence aggregate income is equal to aggregate gross wages in c. Substituting (A.55) back into steps 1-5 yields the equilibrium values for all the endogenous variables of the model.

A.7 Gravity

Bilateral exports from city c to destination n in industry i can be decomposed into the mass of exporting firms times average firm exports:

$$X_{icn} = \left(\frac{1-G_{ic}(\varphi^*_{icn})}{1-G_{ic}(\varphi^*_{icc})}\right) M_{ic} \int_{\varphi^*_{icn}}^{\infty} \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i - 1} \left(\frac{\varphi}{\mu_{ic}}\right)^{\sigma_i - 1} \frac{X_{in}}{(P_{in})^{1-\sigma_i}} (\tau_{icn})^{1-\sigma_i} \frac{dG_{ic}(\varphi)}{1-G(\varphi^*_{icn})},$$

using (A.45) to compute export revenue in n.

Under Pareto productivity, we obtain

$$X_{icn} = M_{ic} \left(\frac{\varphi^*_{icc}}{\varphi_{min,ic}}\right)^{\kappa_i} \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1} \tau_{icn} \mu_{ic}\right)^{1-\sigma_i} \frac{X_{in}}{(P_{in})^{1-\sigma_i}} \left(\frac{\kappa_i}{\kappa_i - \sigma_i + 1}\right) (\varphi_{min,ic})^{\kappa_i} (\varphi^*_{icn})^{\sigma_i - \kappa_i - 1}.$$

We can further simplify this expression using the export cutoff condition (A.47) and the aggregate stability condition (A.51). This yields the standard decomposition of bilateral exports into the extensive and intensive margins of trade,

$$X_{icn} = \frac{M^e_{ic}}{\delta_c} \left(\frac{\varphi_{min,ic}}{\varphi^*_{icn}}\right)^{\kappa_i} \mu_{ic} f_{icn} \left(\frac{\sigma_i - \beta_i}{1-\beta_i}\right) \left(\frac{\kappa_i}{\kappa_i - \sigma_i + 1}\right). \tag{A.56}$$

For estimation purposes, it is convenient to work with the share of exports in sectoral revenue, X_{icF}/R_{ic}, where

$$R_{ic} \equiv \sum_v X_{icv} = \frac{M_{ic}^e}{\delta_c}\mu_{ic}\left(\frac{\sigma_i-\beta_i}{1-\beta_i}\right)\left(\frac{\kappa_i}{\kappa_i-\sigma_i+1}\right)\sum_v\left(\frac{\varphi_{min,ic}}{\varphi_{icv}^*}\right)^{\kappa_i} f_{icv}.$$

Therefore,

$$\frac{X_{icn}}{R_{ic}} = \frac{\left(\frac{\varphi_{min,ic}}{\varphi_{icn}^*}\right)^{\kappa_i} f_{icn}}{\sum_v\left(\frac{\varphi_{min,ic}}{\varphi_{icv}^*}\right)^{\kappa_i} f_{icv}}.$$

Using free entry condition (A.49), the export share simplifies to

$$\frac{X_{icn}}{R_{ic}} = \left(\frac{\sigma_i-1}{\kappa_i-\sigma_i+1}\right)\frac{f_{icn}}{f_{ic}^e}\left(\frac{\varphi_{min,ic}}{\varphi_{icn}^*}\right)^{\kappa_i}.$$

Finally, imposing the export cutoff condition (A.47), we obtain the local gravity equation (17) in the main text,

$$\frac{X_{icn}}{R_{ic}} = \left(\frac{\sigma_i-1}{\kappa_i-\sigma_i+1}\right)\left(\frac{1-\beta_i}{\sigma_i-\beta_i}\right)^{\frac{\kappa_i\sigma_i}{\sigma_i-1}}(\varphi_{min,ic})^{\kappa_i}(f_{icn})^{\frac{\sigma_i-1-\kappa_i}{\sigma_i-1}}(f_{ic}^e)^{-1}(A_{in})^{\frac{\kappa_i}{\sigma_i-1}}(\tau_{icn})^{-\kappa_i}(\mu_{ic})^{\frac{-\kappa_i\sigma_i}{\sigma_i-1}}.$$

A.8 Trade Share

From (A.56), the share of total income of location n spent on goods from city c in industry i can be expressed as:

$$\lambda_{icn} = \frac{X_{icn}}{\sum_v X_{ivn}} = \frac{\delta_c^{-1}M_{ic}^e f_{icn}\mu_{ic}\left(\frac{\varphi_{min,ic}}{\varphi_{icn}^*}\right)^{\kappa_i}}{\sum_v \delta_{iv}^{-1}M_{iv}^e f_{ivn}\mu_{iv}\left(\frac{\varphi_{min,iv}}{\varphi_{ivn}^*}\right)^{\kappa_i}}.$$

Using (A.52),

$$\lambda_{icn} = \frac{\left(\frac{L_{ic}}{\delta_c(1+\delta_c k_{ic})}\right)\left(\frac{f_{icn}}{f_{ic}^e}\right)\mu_{ic}\left(\varphi_{icn}^*\right)^{-\kappa_i}\left(\varphi_{min,ic}\right)^{\kappa_i}}{\sum_v\left(\frac{L_{iv}}{\delta_v(1+\delta_v k_{iv})}\right)\left(\frac{f_{ivn}}{f_{iv}^e}\right)\mu_{iv}\left(\varphi_{ivn}^*\right)^{-\kappa_i}\left(\varphi_{min,iv}\right)^{\kappa_i}}.$$

Using export cutoff conditions, city c's trade share in location n's expenditure on good i can be written as:

$$\lambda_{icn} = \frac{\left(\frac{L_{ic}}{\delta_c(1+\delta_c k_{ic})}\right)\left(f_{ic}^e\right)^{-1}\left(f_{icn}\right)^{1-\frac{\kappa_i}{\sigma_i-1}}\left(\mu_{ic}\right)^{1-\frac{\kappa_i\sigma_i}{\sigma_i-1}}\left(\varphi_{min,ic}\right)^{\kappa_i}\left(\tau_{icn}\right)^{-\kappa_i}}{\sum_v\left(\frac{L_{iv}}{\delta_v(1+\delta_v k_{iv})}\right)\left(f_{iv}^e\right)^{-1}\left(f_{ivn}\right)^{1-\frac{\kappa_i}{\sigma_i-1}}\left(\mu_{iv}\right)^{1-\frac{\kappa_i\sigma_i}{\sigma_i-1}}\left(\varphi_{min,iv}\right)^{\kappa_i}\left(\tau_{ivn}\right)^{-\kappa_i}}.$$

A.9 Welfare

From (A.7), the welfare of the normative representative consumer in any location n can be written as

$$V_n = (\rho)^{-1}\prod_{i=1}^{I}(\alpha_i)^{\alpha_i}\frac{\sum_{i=1}^{I}L_{in}w_{in}}{\prod_{i=1}^{I}(P_{in})^{\alpha_i}},$$

since $W_n = \sum_{i=1}^{I}L_{in}w_{in}$ because aggregate profits are zero (net of entry costs) and net transfers between employed and unemployed workers sum to zero (within cities).

Next, we rewrite the price index (A.53) using (i) the stability condition (A.51) and (A.52) to express the mass of firms as a function of the labor allocation,[4] and (ii) $X_{in} = \alpha_i W_n$, an implication of the Cobb-Douglas assumption under trade balance:

$$P_{in}^{-\kappa_i} = \frac{\left(\frac{\sigma_i-\beta_i}{\sigma_i-1}\right)^{-\kappa_i}\left(\frac{\sigma_i-\beta_i}{1-\beta_i}\right)^{1-\frac{\kappa_i}{\sigma_i-1}}\sum_v\left[\left(\frac{L_{iv}}{\delta_v(1+\delta_v k_{iv})}\right)(f_{iv}^e)^{-1}(\varphi_{min,vi})^{\kappa_i}(\tau_{ivn})^{-\kappa_i}(\mu_{iv})^{1-\frac{\kappa_i\sigma_i}{\sigma_i-1}}(f_{ivn})^{1-\frac{\kappa_i}{\sigma_i-1}}\right]}{(\kappa_i-\sigma_i+1)\left[\frac{1}{1-\beta_i}+\frac{1}{\sigma_i-1}\right](\alpha_i W_n)^{1-\frac{\kappa_i}{\sigma_i-1}}}.$$

In turn, the *domestic* trade share of industry i in location n can be expressed as:

$$\lambda_{inn} = \frac{\left(\frac{L_{in}}{\delta_n(1+\delta_n k_{in})}\right)(f_{in}^e)^{-1}(f_{inn})^{1-\frac{\kappa_i}{\sigma_i-1}}(\mu_{in})^{1-\frac{\kappa_i\sigma_i}{\sigma_i-1}}(\varphi_{min,in})^{\kappa_i}}{\sum_v\left[\left(\frac{L_{iv}}{\delta_v(1+\delta_v k_{iv})}\right)(f_{iv}^e)^{-1}(f_{ivn})^{1-\frac{\kappa_i}{\sigma_i-1}}(\mu_{iv})^{1-\frac{\kappa_i\sigma_i}{\sigma_i-1}}(\varphi_{min,iv})^{\kappa_i}(\tau_{ivn})^{-\kappa_i}\right]}.$$

We can now express the price index as a function of λ_{inn}:

$$P_{in} = \left[\frac{\lambda_{inn}(\alpha_i W_n)^{1-\frac{\kappa_i}{\sigma_i-1}}(\kappa_i-\sigma_i+1)\left[\frac{1}{1-\beta_i}+\frac{1}{\sigma_i-1}\right]}{\left(\frac{\sigma_i-\beta_i}{1-\beta_i}\right)^{1-\frac{\kappa_i}{\sigma_i-1}}\left(\frac{L_{in}}{\delta_n(1+\delta_n k_{in})}\right)(f_{in}^e)^{-1}(f_{inn})^{1-\frac{\kappa_i}{\sigma_i-1}}}\right]^{\frac{1}{\kappa_i}}\frac{(\mu_{in})^{\frac{\sigma_i}{\sigma_i-1}-\frac{1}{\kappa_i}}}{(\varphi_{min,in})}\left(\frac{\sigma_i-\beta_i}{\sigma_i-1}\right). \quad (A.57)$$

Counterfactual changes in Trade Costs. Consider the effects of an arbitrary shock to the vector of variable trade costs, $\{\tau_{ivn}\}$ for any industry i and any two different locations n and v, on the welfare of city c. For any endogenous variable x, let $\dot{x}$ denote the ratio of x after the shock to x before the shock; i.e. the proportional change in the stationary equilibrium value of x.

The proportional change in welfare (real expenditure) in city c is

$$\dot{V}_c = \frac{\dot{W}_c}{\prod_{i=1}^{I}\left(\dot{P}_{ic}\right)^{\alpha_i}}. \quad (A.58)$$

The numerator of (A.58) is given by

$$\dot{W}_c = \dot{e}_c\sum_{i=1}^{I} s_{ic}\dot{\eta}_{ic}\dot{w}_{ic}, \quad (A.59)$$

where $s_{ic} = w_{ic}L_{ic}/W_c$ is industry i's share of income in city c. The denominator follows from (A.57), using $\dot{L}_{ic} = \dot{\eta}_{ic}\dot{e}_c$,

$$\dot{P}_{ic} = \left(\frac{\dot{\lambda}_{icc}}{\dot{\eta}_{ic}}\right)^{\frac{1}{\kappa_i}}\left(\dot{W}_c\right)^{\frac{1}{\kappa_i}-\frac{1}{\sigma_i-1}}(\dot{\mu}_{ic})^{\frac{\sigma_i}{\sigma_i-1}-\frac{1}{\kappa_i}}. \quad (A.60)$$

To simplify (A.59) and (A.60), we use the following key implication of the stationary equilibrium in our model: in every city c, proportional changes in wages and costs of labor are equalized across industries. More formally, for all i and c,

$$\dot{\mu}_{ic} = \dot{w}_{ic} = \dot{g}_c, \quad (A.61)$$

where $\dot{g}_c$ denotes an endogenous city-specific proportional change in μ_{ic} and w_{ic} across industries. For the first equality of (A.61), note that (A.32) implies that the recruitment cost per worker is proportional to the equilibrium wage. Together with (A.39), this implies

$$\mu_{ic} = w_{ic}\left[1+\left(\frac{\rho+\delta_c}{1-\delta_c}\right)k_{ic}\right], \quad (A.62)$$

[4] $M_{iv} = \left(\frac{\varphi_{min,iv}}{\varphi^*_{ivv}}\right)^{\kappa_i}\frac{L_{iv}}{(1+\delta_v k_{iv})\kappa_i f_{iv}^e}\left[\frac{1}{1-\beta_i}+\frac{1}{\sigma_i-1}\right]^{-1}.$

from which $\dot{\mu_{ic}} = \dot{w_{ic}}$ immediately follows. The second equality of (A.61) follows from imposing (A.32) on (A.31) and solving for w_{ic}. This yields

$$w_{ic} = \frac{g_c}{\left[1 - \left(\frac{\beta_i}{1-\beta_i}\right)\left(\frac{\rho+\delta_c}{1-\delta_c}\right) k_{ic}\right]},$$

where g_c is a function of city-specific endogenous variables. Hence. $\dot{w_{ic}} = \dot{g_c}$ for all industries in city c.

Our welfare formula follows from using (A.61) to rewrite (A.59) and (A.60) and then substituting the resulting expressions in (A.58). We thus obtain,

$$\dot{V}_c = (\dot{e}_c)^{1+\sum_{i=1}^{I}\frac{\alpha_i}{\sigma_i - 1}} \left(\sum_{i=1}^{I} s_{ic}\dot{\eta_{ic}}\right)^{1+\sum_{i=1}^{I}\alpha_i\left(\frac{1}{\sigma_i-1}-\frac{1}{\kappa_i}\right)} \prod_{i=1}^{I}\left(\frac{\dot{\lambda}_{icc}}{\dot{\eta}_{ic}}\right)^{-\frac{\alpha_i}{\kappa_i}},$$

Letting $\varepsilon_i = \kappa_i$ and defining Υ_i^e and Υ_i^η as in table 1 of the main text, we obtain the welfare formula (23) for the case MC-FE-HET.

Extension: Trade Liberalization and Labor Endowments. Next, consider the welfare implications of arbitrary changes to the vectors of variable trade costs, $\{\tau_{ivn}\}$, and labor endowments, $\{L_n\}$. The latter capture exogeneous patterns of migration or population growth across locations.

We now focus on the equivalent variation *per-capita* to measure welfare changes in city c, denoted $\dot{V}_c^{PC} \equiv \dot{V}_c/\dot{L}_c$. Proportional changes in per-capita income are still expressed by the right-hand side of (A.59). The change in the local endowment of labor $\dot{L}_c$, however, has an identical impact on domestic expenditure -and, hence, on domestic price indexes- as a change in the employment rate. We thus obtain

$$\dot{V}_c^{PC} = (\dot{e}_c)^{1+\sum_{i=1}^{I}\alpha_i\Upsilon_i^e} \left(\dot{L}_c\right)^{\sum_{i=1}^{I}\alpha_i\Upsilon_i^e} \left(\sum_{i=1}^{I} s_{ic}\dot{\eta_{ic}}\right)^{1+\sum_{i=1}^{I}\alpha_i\Upsilon_i^\eta} \prod_{i=1}^{I}\left(\frac{\dot{\lambda}_{icc}}{\dot{\eta}_{ic}}\right)^{-\frac{\alpha_i}{\varepsilon_i}}. \tag{A.63}$$

This extension of our welfare formula enables the analysis of episodes of economic integration that trigger regional and/or international migration, in addition to changes in trade costs.

A.10 Special Case: Monopolistic Competition, Free Entry and Homogenous Firms (MC-FE-HOM)

In this section, we impose a degenerate productivity distribution. In particular, we assume that the labor productivity of *all* firms in any industry i of any location n is equal to φ_{in}. Moreover, we assume $f_{ivn} = 0 = f_{iv}^e$. Instead, there is a fixed startup cost f_{in} that depends on the industry and location of the producer. Note that, in this setting, all firms in any given cell in export to every destination.

A.10.1 Firm Level Outcomes and Zero-Profit Condition

From (A.46), the profit maximizing price that firms in industry i of city c set in destination n is

$$p_{ivn} = \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1}\right)\frac{\mu_{iv}}{\varphi_{iv}}\tau_{ivn}. \tag{A.64}$$

From (A.45), destination-specific firm revenue can be written as:

$$r_{icn} = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i - 1}\left(\frac{\varphi_{ic}}{\mu_{ic}}\right)^{\sigma_i - 1} A_{in}\left(\tau_{icn}\right)^{1-\sigma_i}. \tag{A.65}$$

Let $r_{ic} \equiv \sum_n r_{icn}$ denote total firm revenue. From (A.40) and (A.42), expected profits upon entry can be expressed as:

$$\Pi_{ic}(0; \varphi_{ic}) = \frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)} \left[r_{ic} \left(\frac{1-\beta_i}{\sigma_i - \beta_i} \right) - \mu_{ic} f_{ic} \right].$$

Due to free entry in any industry i of city c, the zero-profit condition thus requires:

$$r_{ic} \left(\frac{1-\beta_i}{\sigma_i - \beta_i} \right) = \mu_{ic} f_{ic}. \tag{A.66}$$

A.10.2 Gravity

Computing bilateral exports $X_{icn} \equiv M_{ic} r_{icn}$ and sectoral revenue R_{ic}, we obtain the export share:

$$\frac{X_{icn}}{R_{ic}} = \frac{A_{in} (\tau_{icn})^{1-\sigma_i}}{\sum_n A_{in} (\tau_{icn})^{1-\sigma_i}}.$$

Using the zero-profit condition to rewrite the denominator yields

$$\frac{X_{icn}}{R_{ic}} = A_{in} \left(\frac{\tau_{icn}}{\varphi_{ic}} \right)^{1-\sigma_i} \left(\frac{1-\beta_i}{\sigma_i - \beta_i} \right) \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i} \right)^{\sigma_i - 1} (\mu_{ic})^{-\sigma_i} (f_{ic})^{-1}. \tag{A.67}$$

Note that $\sigma_i - 1$ is the trade elasticity.

A.10.3 Labor Demand and Supply

The demand for production workers in cell ic is $M_{ic} l_{ic}$, where firm employment follows from (A.44). Labor demand from fixed costs is simply $M_{ic} f_{ic}$. Again, under (A.32), the industry's demand for recruitment workers is equal to a constant proportion $\delta_c k_{ic}$ of its demand for production and marketing workers. Hence we obtain:

$$L_{ic} = (1+\delta_c k_{ic}) M_{ic} \left[\left(\frac{\sigma_i - 1}{\sigma_i - \beta_i} \right)^{\sigma_i} \frac{(\varphi_{ic})^{\sigma_i - 1}}{(\mu_{ic})^{\sigma_i}} \sum_n A_{in} (\tau_{icn})^{1-\sigma_i} + f_{ic} \right]. \tag{A.68}$$

Using the zero-profit condition (A.66), yields a proportional link between the mass of producers and the mass of workers in the industry:

$$L_{ic} = (1+\delta_c k_{ic}) M_{ic} f_{ic} \left[\frac{\sigma_i - \beta_i}{1-\beta_i} \right]. \tag{A.69}$$

A.10.4 Price Index

The price index is:

$$(P_{in})^{1-\sigma_i} = \sum_v M_{iv} (p_{ivn})^{1-\sigma_i}.$$

Using (A.64), the price index can be expressed as

$$(P_{in})^{1-\sigma_i} = \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1} \right)^{1-\sigma_i} \sum_v M_{iv} \left(\frac{\mu_{iv}}{\varphi_{iv}} \right)^{1-\sigma_i} (\tau_{ivn})^{1-\sigma_i}. \tag{A.70}$$

A.10.5 Trade Share

The trade share is

$$\lambda_{icn} \equiv \frac{X_{icn}}{X_{in}} = \frac{M_{ic} r_{icn}}{\sum_v X_{ivn}}.$$

From revenue (A.65), the domestic trade share can be expressed as

$$\lambda_{inn} = \frac{M_{in} \left(\frac{\varphi_{in}}{\mu_{in}}\right)^{\sigma_i - 1}}{\sum_v M_{iv} \left(\frac{\varphi_{iv}}{\mu_{iv}}\right)^{\sigma_i - 1} (\tau_{ivn})^{1-\sigma_i}}. \tag{A.71}$$

A.10.6 Welfare

Using (A.69) and (A.71), the price index (A.70) in industry i of city c can be written as a function of the domestic trade share:

$$P_{ic} = (\lambda_{icc})^{\frac{1}{\sigma_i - 1}} \left(\frac{L_{ic}}{(1+\delta_c k_{ic}) f_{ic}}\right)^{\frac{1}{1-\sigma_i}} \left[\left(\frac{1-\beta_i}{\sigma_i - \beta_i}\right)\left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i - 1} (\varphi_{ic})^{\sigma_i - 1}\right]^{-1} \mu_{ic}. \tag{A.72}$$

To compute the counterfactual proportional change in the price index, we impose (A.61) and substitute $\dot{L}_{ic} = \dot{\eta}_{ic}\dot{e}_c$ in (A.72). This yields

$$\dot{P}_{ic} = \left(\frac{\dot{\lambda}_{icc}}{\dot{\eta}_{ic}}\right)^{\frac{1}{\sigma_i - 1}} (\dot{e}_c)^{\frac{1}{1-\sigma_i}} \dot{g}_c.$$

The proportional change in aggregate income is still given by (A.59) subject to property (A.61). Substituting the resulting expressions in (A.58) yields

$$\dot{V}_c = (\dot{e}_c)^{1+\sum_{i=1}^{I} \frac{\alpha_i}{\sigma_i - 1}} \left(\sum_{i=1}^{I} s_{ic}\dot{\eta}_{ic}\right) \prod_{i=1}^{I} \left(\frac{\dot{\lambda}_{icc}}{\dot{\eta}_{ic}}\right)^{-\frac{\alpha_i}{\sigma_i - 1}}.$$

Letting $\varepsilon_i = \sigma_i - 1$ and defining Υ_i^e and Υ_i^η as in table 1 of the main text, we obtain the welfare formula (23) for the case MC-FE-HOM.

A.11 Special Case: Monopolistic Competition and Restricted Entry (MC-RE)

In the context of the model of the previous section, here we abandon the free entry condition. In particular, we follow the setup in Arkolakis et al. (2012), where the mass of producers, M_{in}, is fixed and $f_{in} = 0$ for all i and n. A distinct feature of this market structure is that aggregate profits are positive and thus need to be accounted for in the welfare analysis. We assume that profits are distributed back to the representative consumer. Although we focus on the case of homogeneous firms, the derivations below also hold for heterogenous firms with only minor changes.

Aggregate income in city c can be written as:

$$W_c = \sum_{i=1}^{I} \left[L_{ic} w_{ic} + M_{ic} \Pi_{ic}(0, \varphi_{ic})\right], \tag{A.73}$$

where,

$$\Pi_{ic}(0, \varphi_{ic}) = \frac{(1-\delta_c)}{(1+\rho)(\rho+\delta_c)} r_{ic} \left(\frac{1-\beta_i}{\sigma_i - \beta_i}\right),$$

and r_{ic} is defined as in section A.10.1. From (A.68), since now we have $f_{ic} = 0$ for all i and c,

$$L_{ic} = (1 + \delta_c k_{ic}) M_{ic} \sum_n l_{icn}, \tag{A.74}$$

where,

$$l_{icn} = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i} A_{in} (\tau_{icn})^{1-\sigma_i} \frac{(\varphi_{ic})^{\sigma_i - 1}}{(\mu_{ic})^{\sigma_i}}.$$

From (A.65), revenue can be written as a function of l_{icn}:

$$r_{icn} = \left(\frac{\sigma_i - \beta_i}{\sigma_i - 1}\right) \mu_{ic} l_{icn}.$$

Since $r_{ic} = \sum_n r_{icn}$, we can use (A.74) to write aggregate profits as a function of the mass of workers enmployed in cell ic:

$$M_{ic} \Pi_{ic}(0, \varphi_{ic}) = \frac{(1 - \delta_c)(1 - \beta_i)}{(1 + \rho)(\rho + \delta_c)(\sigma_i - 1)(1 + \delta_c k_{ic})} \mu_{ic} L_{ic}.$$

Substituting this equation into aggregate income (A.73) yields

$$W_c = \sum_{i=1}^{I} \left[L_{ic} w_{ic} + \frac{(1 - \delta_c)(1 - \beta_i)}{(1 + \rho)(\rho + \delta_c)(\sigma_i - 1)(1 + \delta_c k_{ic})} \mu_{ic} L_{ic} \right].$$

Under (A.62), the wage w_{ic} is proportional to the cost of labor μ_{ic}. This implies that profits are proportional to labor income in each industry. It is straightforward to verify that proportional changes in the city's aggregate income, $\dot{W}_c$, can still be written as in (A.59), where income shares s_{ic} now account for both rebated profits and labor income.

From (A.70) and (A.71), the price index can be expressed as:

$$(P_{ic})^{1-\sigma_i} = (\lambda_{icc})^{-1} M_{ic} \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right)^{\sigma_i - 1} \left(\frac{\varphi_{ic}}{\mu_{ic}}\right)^{\sigma_i - 1}.$$

Since M_{ic} is fixed, we have

$$\dot{P}_{ic} = \left(\dot{\lambda}_{icc}\right)^{\frac{1}{\sigma_i - 1}} \dot{\mu}_{ic}. \tag{A.75}$$

Substituting (A.75) and (A.59) in (A.58) and imposing property (A.61) yields

$$\dot{V}_c = \dot{e}_c \left(\sum_{i=1}^{I} s_{ic} \dot{\eta}_{ic}\right) \prod_{i=1}^{I} \left(\dot{\lambda}_{icc}\right)^{-\frac{\alpha_i}{\sigma_i - 1}}$$

Letting $\varepsilon_i = \sigma_i - 1$ (we show this next) and defining Υ_i^e and Υ_i^η as in table 1 of the main text, we obtain the welfare formula (23) for the case MC-RE.

A.11.1 Gravity with Restricted Entry

As in the previous section,

$$\frac{X_{icn}}{R_{ic}} = \frac{A_{in} (\tau_{icn})^{1-\sigma_i}}{\sum_{n'} A_{in'} (\tau_{icn'})^{1-\sigma_i}}.$$

From (A.74),

$$\left[\sum_{n'} A_{in} (\tau_{icn})^{1-\sigma_i}\right]^{-1} = \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right) \frac{M_{ic}}{L_{ic}} (\varphi_{ic})^{\sigma_i - 1} (\mu_{ic})^{-\sigma_i}.$$

Hence we obtain:

$$\frac{X_{icn}}{R_{ic}} = A_{in} \left(\frac{\sigma_i - 1}{\sigma_i - \beta_i}\right) \frac{M_{ic}}{L_{ic}} (\varphi_{ic})^{\sigma_i - 1} (\tau_{icn})^{1-\sigma_i} (\mu_{ic})^{-\sigma_i}.$$

A.12 Perfect Competition (PC)

A.12.1 An Armington Model with Multiple Sectors and Search Frictions

In this section, we assume that goods are differentiated by country of origin. In particular, each country produces a unique variety of every good i under constant returns to labor. Labor productivity, denoted φ_{ic}, varies arbitrarily across industries and cities. Markets for goods are now perfectly competitive but the structure of local labor markets remains unchanged, as described in the main text. Firms incur no marketing expenses because entry and fixed costs are assumed to be zero. Consumer preferences are still given by (A.1), although here we interpret the integral sign as a Lebesgue integral to allow for a finite set of varieties in each industry.

A.12.2 Production

Under constant returns to labor, the value functions of firms in any given industry-city cell are independent of the number of production workers employed. Without loss of generality, we can therefore assume that all firms are single-worker firms (or jobs).

In a stationary equilibrium, the expected profit of a firm in industry i of city c upon entry are described by special cases of (A.37) and (A.38); in particular,

$$\Pi_{ic}(0;\varphi_{ic}) = \frac{1}{1+\rho}\left[-\frac{p_{ic}^V}{m_c(\theta_c)} + (1-\delta_c)\,\Pi_{ic}(\varphi_{ic})\right],$$

where

$$\Pi_{ic}(\varphi_{ic}) = \frac{1}{1+\rho}\left[p_{ic}\varphi_{ic} - w_{ic} + (1-\delta_c)\,\Pi_{ic}(\varphi_{ic})\right] \tag{A.76}$$

and p_{ic} denotes the competitive factory gate price.

Imposing the zero-profit condition, $\Pi_{ic}(0,\varphi_{ic}) = 0$ and manipulating the value functions yields

$$p_{ic}\varphi_{ic} = \mu_{ic}, \tag{A.77}$$

where the cost of labor μ_{ic} is defined as in (A.39).

A.12.3 Wage Bargaining

As anticipated, the structure of the labor market is unchanged. Under perfect competition in the goods market, however, the surplus-splitting rule is now written as

$$(1-\beta_i)\left[E_{ic} - U_c\right] = \beta_i \Pi_{ic}(\varphi_{ic}), \tag{A.78}$$

where $\Pi_{ic}(\varphi_{ic})$ is obtained from equation (A.76). Substituting the value functions (A.25) and (A.30), and imposing fiscal and trade balance as in section A.10.1 yields equation (A.31).

Assuming a constant recruitment cost per worker, (A.32), we obtain (A.61), the key property that ensures tractability in the counterfactual welfare analysis. Combining the latter with (A.77), we have

$$\dot{p_{ic}} = \dot{\mu_{ic}} = \dot{w_{ic}} = \dot{g_c}. \tag{A.79}$$

A.12.4 Gravity

Trade is subject to iceberg variable trade costs only. Due to the structure of preferences, this implies that each location exports to all destinations in any given industry. The value of sales of goods from city c to location n in industry i is $X_{icn} = p_{ic}\tau_{icn}q_{icn}$. Using the CES demand and $p_{ic}\varphi_{ic} = \mu_{ic}$ yields the gravity equation in terms of the cost of labor; that is,

$$X_{icn} = \tau_{icn}^{1-\sigma_i}\mu_{ic}^{1-\sigma_i}\varphi_{ic}^{\sigma_i-1}X_{in}P_{in}^{\sigma_i-1},$$

where X_{in} denotes location n's expenditure on industry i. Therefore, the trade and cost-of-labor elasticities are both equal to $\sigma_i - 1$. Note, however, that the export share is independent of the cost of labor μ_{ic} conditional on the demand shifter in the destination.

To derive our welfare formula below, we work with the domestic trade share expressed in terms of the factory gate price. Again, using the CES functional form yields

$$\lambda_{icc} \equiv \frac{X_{icc}}{X_{ic}} = \left(\frac{p_{ic}\tau_{icc}}{P_{ic}}\right)^{1-\sigma_i}. \tag{A.80}$$

A.12.5 Welfare

Again, we consider an arbitrary shock to the vector of trade costs that leaves domestic trade costs unchanged; i.e. $\dot{\tau_{icc}} = 1$ for all c.

The welfare function is still given by (A.58), where the numerator satisfies (A.59). For the denominator, we solve for the change in the price index from (A.80). Imposing property (A.79) yields

$$\dot{V_c} = \dot{e_c}\left(\sum_{i=1}^{I} s_{ic}\dot{\eta_{ic}}\right)\prod_{i=1}^{I}\left(\dot{\lambda_{icc}}\right)^{-\frac{\alpha_i}{\sigma_i - 1}}.$$

Letting $\varepsilon_i = \sigma_i - 1$ and defining Υ_i^e and Υ_i^η as in table 1 of the main text, we obtain the welfare formula (23) for the case PC.

B Worker Mobility

In the model presented in the text, we have assumed that workers are not mobile across cities. This assumption, however, is not essential for the result that local industrial composition matters for wages – which is the basis for our identification strategy – and allows us to greatly simplify the exposition of the main elements of our framework. However, since worker mobility across local labor markets seems like a natural margin of adjustment to local shocks, we briefly discuss how this margin could be incorporated into our model. First, suppose that unemployed workers were occasionally offered the option to move to a different local labor market and, once given this option, could choose the local labor market c' that maximized their indirect utility. This extension would add an additional term to the equation for the value of unemployed search, U_c, capturing the value of this mobility option: $m \cdot \max\{(U_{c'} - \theta_{cc'}^m - U_c), 0\}$, where m is the probability that a worker is offered the mobility option and $\theta_{cc'}^m$ is the location-specific cost of moving to c'.

If the option to move occurs frequently enough (m is sufficiently high), a steady state spatial equilibrium will imply that the value of moving is driven to zero. When incorporating mobility, as in standard spatial equilibrium models, it is appropriate to include housing (or land) costs as an equilibrating mechanism. These costs do not influence wage bargaining, because they have to be incurred whether or not a worker is employed (and wages depend on the difference $[E_{ic}(l;\varphi) - U_c]$). However, housing prices will have to adjust to equate expected utility across locations. Assuming that housing is not perfectly elastically supplied, an improvement of industrial composition in c will make c more attractive and there will be an inflow of workers from other locations. This will simultaneously increase the cost of housing in c and this process will continue until $m \cdot \max\{(U_{c'} - \theta_c^m - U_c), 0\}$ is driven to zero but before wages are equalized. Thus, as long as the mobility friction is small enough, the option to move is *directed*: unemployed workers decide to search locally or move to c' and search.[5] A spatial equilibrium implies that workers must be indifferent between these options. However, if mobility frictions are large, it will prevent the spatial indifference. In this scenario, mobility frictions,

[5]Random search across cities has no impact on our framework because the outside option in random search doesn't depend on c, and thus can be captured by an intercept in an empirical specification.

importantly, will still imply that local industrial composition is a determinant of local wages, but wages will also be influenced by an additional outside option to move.[6]

In order to simplify the exposition of our model, we assume that mobility frictions are sufficiently low in order for a spatial equilibrium to hold, and thus we ignore the mobility option. We will, however, examine the relevance of mobility as an adjustment mechanism in section 6 of our paper where we examine the impact of trade shocks on local labor market outcomes. To preview those results, we find no evidence that the observed trade shocks over the period that we study had any effect on population sizes of local labor markets. This result is consistent with Autor et al. (2013) for the U.S. and Dauth et al. (2014) for Germany, who also find minimal population adjustments to trade shocks.

C Empirical Gravity and Domestic Revenue Equations

In this section we derive the empirical gravity and domestic revenue equations, and provide explicit details on the structure of the error term and its implications for the estimation of ϕ^G and ϕ^R.

Gravity Equation. Substituting equations (11) and (12) into equation (17) in the main text, we obtain:

$$\frac{X_{icF}}{R_{ic}} = \Lambda_i^1 \left(f_{ic}^E\right)^{-1} \left(f_{icF}\right)^{1-\frac{\varepsilon}{\sigma-1}} \left(\frac{\varphi_{\min,ic}}{\tau_{icF}}\right)^{\varepsilon} \left(A_{iF}\right)^{\frac{\varepsilon}{\sigma-1}} \left(1+\frac{\rho+\delta_c}{1-\delta_c}k_{ic}\right)^{-\frac{\varepsilon\sigma}{\sigma-1}} w_{ic}^{-\frac{\varepsilon\sigma}{\sigma-1}}, \tag{C.1}$$

where we let $\sigma_i = \sigma$ and $\varepsilon_i = \varepsilon$, as described in the main text. Taking logs and adding the time subscript, equation (C.1) rewrites as:

$$\ln \frac{X_{icFt}}{R_{ict}} = \ln \Lambda_{it}^1 + \frac{\varepsilon}{\sigma-1} \ln A_{iFt} - \frac{\varepsilon\sigma}{\sigma-1} \ln w_{ict} + \tilde{u}_{ict}^G, \tag{C.2}$$

where

$$\tilde{u}_{ict}^G = -\ln f_{ict}^E + \left(1-\frac{\varepsilon}{\sigma-1}\right) \ln f_{icFt} + \varepsilon \ln \varphi_{\min,ict} - \varepsilon \ln \tau_{icFt} - \frac{\varepsilon\sigma}{\sigma-1} \ln\left(1+\frac{\rho+\delta_c}{1-\delta_c}k_{ict}\right) \tag{C.3}$$

The first two terms of equation (C.2) are industry-year specific and depend on the parameter Λ_{it}^1, the foreign demand shifter (A_{iFt}) and the elasticities σ and ε. The last term, $\tilde{u}_{ict}^G$, varies across industries, cities and years. As can been seen from equation (C.3), it is function of the fixed costs (f_{ict}^E and f_{icFt}), the iceberg cost (τ_{icFt}), the lower-bound of the productivity distribution ($\varphi_{\min,ict}$), the recruitment cost shifter (as captured by k_{ict}) and the elasticities σ and ε.

In order to move to the empirical gravity equation, it is useful to decompose $\tilde{u}_{ict}^G$ into an industry-year component and a residual industry-city-year component; i.e. $\tilde{u}_{ict}^G = u_{it}^G + u_{ict}^G$, where u_{it}^G is the mean of $\tilde{u}_{ict}^G$ across cities and the residual component u_{ict}^G captures the deviation of $\tilde{u}_{ict}^G$ from the mean. In particular, note that this residual component is given by:

[6] Moreover, the forces we emphasize in our model also have implications for worker mobility and housing costs. See for example, Beaudry et al. (2012, 2014); Green et al. (2017) who examine worker mobility in a setting of frictional local labor markets.

$$
\begin{aligned}
u^G_{ict} &= -\left(\ln f^E_{ict} - \overline{\ln f^E_{ict}}\right) + \left(1 - \frac{\varepsilon}{\sigma-1}\right)\left(\ln f_{icFt} - \overline{\ln f_{icFt}}\right) + \varepsilon\left(\ln \varphi_{\min,ict} - \overline{\ln \varphi_{\min,ict}}\right) \\
&- \varepsilon\left(\ln \tau_{icFt} - \overline{\ln \tau_{icFt}}\right) - \frac{\varepsilon\sigma}{\sigma-1}\left[\ln\left(1 + \frac{\rho+\delta_c}{1-\delta_c}k_{ict}\right) - \overline{\ln\left(1 + \frac{\rho+\delta_c}{1-\delta_c}k_{ict}\right)}\right] \\
&= -\widehat{\ln f^E_{ict}} + \left(1 - \frac{\varepsilon}{\sigma-1}\right)\widehat{\ln f_{icFt}} + \varepsilon \widehat{\ln \varphi_{\min,ict}} - \varepsilon \widehat{\ln \tau_{icFt}} - \frac{\varepsilon\sigma}{\sigma-1}\widehat{\ln\left(1 + \frac{\rho+\delta_c}{1-\delta_c}k_{ict}\right)},
\end{aligned}
\tag{C.4}
$$

where for any model parameter x, $\overline{x}$ denotes the mean of x across cities, and $\hat{x}$ denotes the deviation of x from the mean. Thus, this residual term is a collection of deviations of log model parameters from their industry-specific mean. Each deviation can be thought of as capturing an industry-city-specific cost comparative advantage.

Using the decomposition of $\tilde{u}^G_{ict}$ and first-differencing over time, equation (C.2) further rewrites as:

$$
\Delta \ln \frac{X_{icFt}}{R_{ict}} = \Delta \ln \Lambda^1_{it} + \frac{\varepsilon}{\sigma-1}\Delta \ln A_{iFt} + \Delta u^G_{it} - \frac{\varepsilon\sigma}{\sigma-1}\Delta \ln w_{ict} + \Delta u^G_{ict}, \tag{C.5}
$$

where Δ denotes a difference. The first three terms can be entirely controlled for using industry-year fixed effects. Hence, we arrive at equation (24) in the main text:

$$
\Delta \ln \frac{X_{icFt}}{R_{ict}} = \Delta d^G_{it} + \phi^G \Delta \ln w_{ict} + \Delta u^G_{ict}, \tag{C.6}
$$

where $\Delta d^G_{it} = \Delta \ln \Lambda^1_{it} + \frac{\varepsilon}{\sigma-1}\Delta \ln A_{iFt} + \Delta u^G_{it}$ and $\phi^G = -\frac{\varepsilon\sigma}{\sigma-1}$. Finally, the error term Δu^G_{ict} is given by the first difference of equation (C.4) and captures changes in industry-city unobservable comparative advantages.

Consistent estimation of ϕ^G requires that $\mathbf{E}[\Delta \ln w_{ict} \Delta u^G_{ict}] = 0$. Given the structure of the error term and the fact that the elasticities σ and ε are constant (independent of the city), the expectation can be written as:

$$
\begin{aligned}
\mathbf{E}[\Delta \ln w_{ict} \Delta u^G_{ict}] &= -\mathbf{E}\left[\Delta \ln w_{ict} \Delta \widehat{\ln f^E_{ict}}\right] + \left(1 - \frac{\varepsilon}{\sigma-1}\right)\mathbf{E}\left[\Delta \ln w_{ict} \Delta \widehat{\ln f_{icFt}}\right] + \varepsilon \mathbf{E}\left[\Delta \ln w_{ict} \Delta \widehat{\ln \varphi_{\min,ict}}\right] \\
&- \varepsilon \mathbf{E}\left[\Delta \ln w_{ict} \Delta \widehat{\ln \tau_{icFt}}\right] - \frac{\varepsilon\sigma}{\sigma-1}\mathbf{E}\left[\Delta \ln w_{ict} \Delta \widehat{\ln\left(1 + \frac{\rho+\delta_c}{1-\delta_c}k_{ict}\right)}\right]
\end{aligned}
\tag{C.7}
$$

Therefore, the structure of the error term makes it clear that a sufficient condition for $\mathbf{E}[\Delta \ln w_{ict} \Delta u^G_{ict}] = 0$ is that variation in industry-city log wages are uncorrelated to shocks to each of the cost comparative advantages; i.e. to shocks to the deviation from the mean of the log fixed costs (i.e., formally, $\frac{1}{C}\sum_c \Delta \ln w_{ict} \Delta \widehat{\ln f^E_{ict}} \to^p \mathbf{E}\left[\Delta \ln w_{ict} \Delta \widehat{\ln f^E_{ict}}\right] = 0$ as $C \to \infty$), iceberg cost, productivity threshold, and the log function of the recruitment cost shifter k_{ict}.

As discussed in the main text, this condition cannot hold since, under search-and-bargaining frictions, wages necessarily depend on shifts in the recruitment cost shifter. Therefore, ordinary least squares would yield inconsistent estimates of ϕ^G and this is why we turn to an instrumental variable strategy.

Domestic Revenue Equation. Substituting equations (11) and (12) into equation (13) in the main text, and setting $I_{icn}(\varphi) = 1$ for $n \neq F$ and zero otherwise, we obtain:

$$r_{ic}(\varphi) = \left(\frac{\sigma-1}{\sigma-\beta_i}\right)^{\sigma-1} \tilde{A}_{ic}\varphi^{\sigma-1}\left(1+\frac{\rho+\delta_c}{1-\delta_c}k_{ic}\right)^{1-\sigma} w_{ic}^{1-\sigma}, \tag{C.8}$$

where, as before, we let $\sigma_i = \sigma$ and $\varepsilon_i = \varepsilon$, and where $\tilde{A}_{ic} = \sum_{n\neq F} A_{ic}(\tau_{icn})^{1-\sigma}$. Taking logs and adding the time subscript, equation (C.1) rewrites as:

$$\ln r_{ict}(\varphi) = (\sigma-1)\ln\left(\frac{\sigma-1}{\sigma-\beta_i}\right) + (1-\sigma)\ln w_{ict} + \ln\tilde{A}_{ict} + (\sigma-1)\ln\varphi_t + \tilde{u}^R_{ict}, \tag{C.9}$$

where

$$\tilde{u}^R_{ict} = (1-\sigma)\ln\left(1+\frac{\rho+\delta_c}{1-\delta_c}k_{ic}\right) \tag{C.10}$$

Decomposing $\tilde{u}^R_{ict}$ into a mean component u^R_{it} and a residual component u^R_{ict}, as we did for $\tilde{u}^R_{ict}$, and first-differencing over time we obtain:

$$\Delta\ln r_{ict}(\varphi) = \Delta u^R_{it} + (1-\sigma)\Delta\ln w_{ict} + (\sigma-1)\Delta\ln\varphi_t + \Delta\ln\tilde{A}_{ict} + \Delta u^R_{ict}, \tag{C.11}$$

where $\Delta u^R_{ict} = (1-\sigma)\Delta\ln\left(\widehat{1+\frac{\rho+\delta_c}{1-\delta_c}k_{ic}}\right)$ is the error term. Unlike Δu^G_{ict}, the error term only depends on the recruitment cost shifter and the elasticity. The first term term of equation (C.11) can be captured by a set of industry-year FE and the third term is controlled for by firm FE. Substituting in these FE, we arrive at equation (25) in the main text:

$$\Delta\ln r_{ict}(\varphi) = \Delta d^R_{it} + \phi^R\Delta\ln w_{ict} + T(\varphi) + \Delta\ln\tilde{A_{ict}} + \Delta u^R_{ict}(\varphi), \tag{C.12}$$

where $\phi^R = 1-\sigma$, Δd^R_{it} denotes industry-year FE and $T(\varphi)$ firm FE.

Consistent estimation of ϕ^R requires that $\mathbf{E}[\Delta\ln w_{ict}\Delta u^R_{ict}] = 0$. Given the structure of the error term and the fact that the elasticities σ and ε are constant (independent of the city), a sufficient condition for $\mathbf{E}[\Delta\ln w_{ict}\Delta u^R_{ict}] = 0$ is that variation in industry-city log wages are uncorrelated to shocks to the log function of the recruitment cost shifter k_{ict}; specifically, that $\frac{1}{C}\sum_c \Delta\ln w_{ict}\Delta\ln\left(\widehat{1+\frac{\rho+\delta_c}{1-\delta_c}k_{ic}}\right) \to^p \mathbf{E}\left[\Delta\ln w_{ict}\Delta\ln\left(\widehat{1+\frac{\rho+\delta_c}{1-\delta_c}k_{ic}}\right)\right] = 0$ as $C\to\infty$. As for the gravity equation, this condition is necessarily violated since the wage equation depends on the recruitment cost shifter as well. For this reason, we also use instruments when estimating the domestic revenue equation.

D Derivation of the Wage Equation

The goal of the linear approximation is to link the industry-city wage to the industrial composition of the local labor market. To simplify the exposition, we impose constant exit rates and bargaining power, i.e. $\delta_c = \delta$ and $\beta_i = \beta$. Substituting equation (A.29) in (A.30), using equation (A.33) and solving for the industry-city wage yields:

$$w_{ic} = \tilde{k}_{ic}\cdot\left[(1-\tilde{\gamma}_{2c})(t_c+b_c) + \tilde{\gamma}_{2c}\bar{w}_c\right], \tag{D.1}$$

where $\bar{w}_c = \sum_i \eta_{ic}w_{ic}$, $\tilde{\gamma}_{2c} = \left[\frac{\theta_c m_c(\theta_c)}{\rho+\delta+\theta_c m_c(\theta_c)}\right]$, and the parameter $\tilde{\gamma}_{2c}$ is dependent on the tightness of the labor market. The parameter $\tilde{k}_{ic} = \left[\frac{(1-\beta)(1-\delta)}{(1-\beta)(1-\delta)-\beta(\rho+\delta)k_{ic}}\right]$ is a function of the recruitment cost shifter k_{ic}.

To further simplify equation (D.1), we rewrite the labor tax as a function of the unemployment benefits. Using the fiscal balance condition $(1-e_c)b_c = e_c t_c$ together with the Beveridge curve $(1-e_c)\theta_c m_c(\theta_c) = \delta e_c$, the labor tax is given by:

$$t_c = \frac{\delta}{\theta_c m_c(\theta_c)} b_c. \tag{D.2}$$

Substituting (D.2) into equation (D.1) we obtain:

$$w_{ic} = \tilde{k}_{ic} \cdot (\tilde{\gamma}_{1c} b_c + \tilde{\gamma}_{2c} \bar{w}_c), \tag{D.3}$$

where $\tilde{\gamma}_{1c} = (1-\tilde{\gamma}_{2c})\left[\frac{\delta+\theta_c m_c(\theta_c)}{\theta_c m_c(\theta_c)}\right]$. Solving for $\bar{w}_c$, we obtain:

$$\bar{w}_c = \frac{\bar{\tilde{k}}_{ic}\tilde{\gamma}_{1c} b_c}{1-\bar{\tilde{k}}_{ic}\tilde{\gamma}_{2c}}, \tag{D.4}$$

where $\bar{\tilde{k}}_{ic} = \sum_i \eta_{ic}\tilde{k}_{ic}$, and substituting back into equation (D.3), the reduced-form wage equation can be written as:

$$w_{ic} = \tilde{k}_{ic}\tilde{\gamma}_{1c} b_c \left(\frac{1}{1-\bar{\tilde{k}}_{ic}\tilde{\gamma}_{2c}}\right), \tag{D.5}$$

In order to take the linear approximation of w_{ic}, it is useful to decompose $\tilde{k}_{ic}$, without loss of generality, as follows:

$$\tilde{k}_{ic} = \tilde{k}_i + \tilde{k}_c + \tilde{\xi}_{ic},$$

where $\tilde{k}_i$ represents a common (across cities) industry component, $\tilde{k}_c$ represents city-specific component, and ξ_{ic} is an idiosyncratic component that sums to zero across industries, within cities (i.e. $\sum_i \xi_{ic} = 0$).

Using this decomposition of $\tilde{k}_{ic}$, one can rewrite the wage equation as:

$$w_{ic} = (\tilde{k}_i + \tilde{k}_c + \tilde{\xi}_{ic})\tilde{\gamma}_{1c} b_c \left[\frac{1}{1-\tilde{\gamma}_{2c}(\tilde{K}_c + \tilde{k}_c + \sum_i \eta_{ic}\tilde{\xi}_{ic})}\right],$$

where $\tilde{K}_c = \sum_i \eta_{ic}\tilde{k}_i$ captures the weighted city-average of the national-level component of $\tilde{k}_{ic}$.

Let $w_{ic} = w_{ic}(b_c, \tilde{K}_c, \tilde{k}_i, \tilde{k}_c, \{\xi_{ic}\}_i, e_c)$ describe the reduced-form equation. We take a linear approximation of the wage equation around the point where recruitment cost shifters are constant and the employment rate is the same across cities, i.e. around $x_0 = (b_c = b_0, \tilde{K}_c = \tilde{k}^i, \tilde{k}_i = \tilde{k}^i, \tilde{k}_c = \tilde{k}^c, \{\tilde{\xi}_{ic}\}_i = 0, e_c = e_0)$. Around that point, cities have an identical industrial composition (i.e. $\eta_{ic} = 1/I$). The log-linear approximation is given by:

$$\ln w_{ic} \quad \approx \quad \gamma_0 + \gamma_1 b_c + \gamma_2 \sum_i \eta_{ic} k_i + \gamma_3 k_i + \gamma_4 k_c + \gamma_5 e_c + \gamma_3 \xi_{ic}, \tag{D.6}$$

where, using $\overline{w}$ as an arbitrary constant, writing $k_i = \frac{\tilde{k}_i}{\overline{w}}$, $k_c = \frac{\tilde{k}_c}{\overline{w}}$, $\xi_{ic} = \frac{\tilde{\xi}_{ic}}{\overline{w}}$, and using the property that $\sum_i \tilde{\xi}_{ic} = 0$, we have:

$$\ln w_{ic} \approx \ln \overline{w} + \frac{(w_{ic} - \overline{w})}{\overline{w}},$$
$$\gamma_0 = \ln \overline{w} + \frac{(\tilde{\gamma}_0 - \overline{w})}{\overline{w}},$$
$$\gamma_1 = \frac{1}{\overline{w}} \cdot \Upsilon \cdot (\tilde{k}^i + \tilde{k}^c)\tilde{\gamma}_1,$$
$$\gamma_2 = \Psi^2 \cdot (\tilde{k}^i + \tilde{k}^c)\tilde{\gamma}_1 \tilde{\gamma}_2 b_0,$$
$$\gamma_3 = \Psi \cdot \tilde{\gamma}_1 b_0,$$
$$\gamma_4 = \gamma_2 + \gamma_3,$$
$$\gamma_5 = \frac{1}{\overline{w}} \cdot \Psi \left[\frac{\partial \tilde{\gamma}_{1c}}{\partial e_c}\Big|_{x_0} + \tilde{\gamma}_1 (\tilde{k}^i + \tilde{k}^c) \Psi \frac{\partial \tilde{\gamma}_{2c}}{\partial e_c}\Big|_{x_0} \right] \cdot (\tilde{k}^i + \tilde{k}^c) \cdot b_0,$$
$$\Psi = \left[\frac{1}{1 - \tilde{\gamma}_2 (\tilde{k}^i + \tilde{k}^c)} \right],$$
$$\tilde{\gamma}_0 = -\Psi(\tilde{k}^i + \tilde{k}^c) b_0 \left\{ \tilde{\gamma}_1 + e_0 \left[\frac{\partial \tilde{\gamma}_{1c}}{\partial e_c}\Big|_{x_0} + \tilde{\gamma}_1 (\tilde{k}^i + \tilde{k}^c) \Psi \frac{\partial \tilde{\gamma}_{2c}}{\partial e_c}\Big|_{x_0} \right] \right\},$$

and where $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ are constant parameters corresponding to $\tilde{\gamma}_{1c}$ and $\tilde{\gamma}_{2c}$, evaluated at x_0, respectively.

Importantly, equation (D.6) shows that, at the national level, inter-industry wage differentials are given by $\gamma_3 k_i$, which expresses the average wage in industry i relative to an omitted group. Letting $\nu_i = \gamma_3 k_i$ denote the national industry wage premium, we finally express log industry-city wages as a function of industrial composition:

$$\ln w_{ic} = \gamma_0 + \gamma_1 b_c + \frac{\gamma_2}{\gamma_3} K_c + \gamma_3 k_i + \gamma_4 k_c + \gamma_5 e_c + \epsilon_{ict}, \tag{D.7}$$

where $K_c = \sum_i \eta_{ic} \nu_i$ is an index that captures industrial composition at the city level, and $\epsilon_{ict} = \gamma_3 \xi_{ic} + \tilde{\epsilon}_{ict}$ where $\tilde{\epsilon}_{ict}$ is an error from the approximation.

E Details of Instrument Construction

In order to construct our instruments, IV_{ct}^W and IV_{ct}^B, we need (1) estimates of the national industrial premia (since ν_{it} are unobserved) and (2) to predict local industrial employment shares (since η_{ict} are potentially correlated with the error terms in (24) and (25)). We discuss these steps below.

Estimating National Wage Premia. Equation (27) shows that wages vary because of an industry-specific component (ν_{it}), a city-specific component ($\gamma_0 + \gamma_1 b_{ct} + \frac{\gamma_2}{\gamma_3} K_{ct} + \gamma_4 k_{ct} + \gamma_5 e_{ct}$) and an idiosyncratic term ($\gamma_3 \xi_{ict}$). An implication is that the inclusion of a set of city fixed effects in a wage regression at the industry-city level would allow us to recover national industrial wage premia from the estimated coefficients on industry fixed effects, without directly observing K_{ct}, and the local component of the vacancy posting cost, k_{ct}.

However, in order to take the model's wage equation to the data, we must confront the fact that workers are heterogeneous in our data but not in the model. Our approach is to treat individuals as representing different bundles of efficiency units of labor, and assume these bundles are perfect substitutes in production. We interpret w_{ict} in (27) as the cost per effective labor unit and index worker characteristics by H_h. Let effective labor units be $\exp(H_n' \mathscr{B} + a_h)$, where H_h and a_h capture observable and unobservable skills of worker h, respectively. Adding industry, city and time subscripts, workers log wages, $\ln W_{hict}$, are given by:

$$\ln W_{hict} = H'_{ht}\mathscr{B}_t + \ln w_{ict} + a_{hict}. \tag{E.8}$$

We estimate (E.8) separately by year, while capturing $\ln w_{ict}$ with a complete set of industry-city dummies. We interpret the estimated vector coefficients on the city-industry fixed-effects as regression-adjusted city-industry average wages, which we denote by $\widehat{\ln w_{ict}}$. In practice, H'_{ht} includes age, the square of age, a gender dummy, a nationality dummy, a categorical variable for education and a full set of education-gender, education-nationality and education-age interactions.

Pooling across years, we then estimate an empirical version of (27), regressing $\widehat{\ln w_{ict}}$ on a set of city-year and industry-year fixed effects. The inclusion of the city-year fixed effects absorbs local economic conditions given by $\gamma_0 + \gamma_1 b_{ct} + \frac{\gamma_2}{\gamma_3} K_{ct} + \gamma_4 k_{ct} + \gamma_5 e_{ct}$ in equation (27) and the coefficients on the industry-year fixed-effects estimate the national-level industrial wage differentials, $\hat{\nu}_{it}$.

Predicting Shares. We predict local employment shares by combining estimates of national-level industrial growth with base-period local industrial composition. Since we have many industries within each city-year, we pursue a generalized leave-one-out method that purges a common city component from the national-level industry growth. The procedure that we use closely follows Greenstone et al. (2020). Consider the following equation for local industry-city employment growth:

$$\Delta \ln L_{ict} = \mathcal{G}_{it} + \mathcal{G}_{ct} + \tilde{\mathcal{G}}_{ict}, \tag{E.9}$$

where $\mathcal{G}_{ct}$ are city-time fixed effects and $\mathcal{G}_{it}$ are industry-year effects. This equation describes local industry employment growth as stemming from national-level factors common across cities ($\mathcal{G}_{it}$), city-level factors that are common across industries ($\mathcal{G}_{ct}$), and an idiosyncratic city-industry factor ($\tilde{\mathcal{G}}_{ict}$). The inclusion of $\mathcal{G}_{ct}$ is meant to absorb growth due to conditions in the local economy, such as demand shocks.

The vector of coefficients on the $\mathcal{G}_{it}$ fixed-effects are associated with national-level forces. We use their estimates, denoted $\hat{\mathcal{G}}_{it}$, to predict local industry size based on local base-period employment:

$$\hat{L}_{ict} = L_{ic0} \prod_{s=1}^{t} \left(1 + \hat{\mathcal{G}}_{is}\right),$$

for $t \geq 1$, where L_{ic0} is a base-period level of employment in industry i in the local economy c. We then convert predicted employment into shares, as discussed in the main text.

Constructing Instruments. With $\hat{\eta}_{jct}$ and $\hat{\nu}_{jt}$ at hand, we construct instruments:

$$IV^W_{ct} = \sum_{j \in S} \hat{\eta}_{jct-1} \Delta \hat{\nu}_{jt},$$
$$IV^B_{ct} = \sum_{j \in S} \hat{\nu}_{jt} \Delta \hat{\eta}_{jct},$$

where $\hat{\eta}_{jct}$ are only functions of base period shares and national growth rates and S denotes the set of non-manufacturing industries. In practice, to predict local industry size, we average industry-city employment over the period 1992-1993, i.e. $L_{ic0} = (L_{ic1992} + L_{ic1993})/2$. We then leave one year out and first predict employment, $\hat{L}_{ict}$, in 1995, which restricts our sample to the period 1996-2010 since IV^B_{ct} and IV^W_{ct} both use $t-1$ predicted employment shares.

F Consistency, Heterogeneity and the Over-identification Test

In this section we discuss the conditions under which our instrumental variable strategy is valid, interpretation of our estimates under heterogeneity, and the over-identification test discussed in the main text.

To begin, consider the sample covariance between our within-instrument and the error of the gravity equation:

$$\frac{1}{I}\frac{1}{C}\sum_{c}\sum_{i\notin S}\left(\sum_{j\in S}\hat{\eta}_{jct-1}\Delta\nu_{jt}\right)\Delta u_{ict}^{G}=\frac{1}{I}\frac{1}{C}\sum_{i\notin S}\sum_{j\in S}\Delta\nu_{jt}\sum_{c}\hat{\eta}_{jct-1}\Delta u_{ict}^{G} \tag{F.10}$$

where the last summation on the right-hand-side is the city-level covariance between predicted non-manufacturing shares and the error term. Predicted shares are only a function of base-period non-manufacturing industrial composition and national-level industrial growth rates. The error term contains changes in the residual component of a number of model parameters, and can be generally interpreted in our framework as changes in city-industry unobservable comparative advantage. A sufficient condition for consistency is that $\frac{1}{C}\sum_c \hat{\eta}_{jct-1}\Delta u_{ict}^{G} \to^p \mathbf{E}[\hat{\eta}_{jct-1}\Delta u_{ict}^{G}]=0$ as $C\to\infty$. In words, if base-period non-manufacturing industrial composition does not predict future *changes* in comparative advantage in manufacturing industries, our instrument is valid. Thus, our instruments would be valid, for example, under a random-walk type assumption for Δu_{ict}^{G}, as emphasized in Beaudry et al. (2012, 2018).

Recently, Borusyak et al. (2018) have shown that even if this condition breaks down, Bartik-style instruments may still be valid. In their paper, they emphasize the conditions under which $\frac{1}{C}\sum_c \hat{\eta}_{jct-1}\Delta u_{ict}^{G}$ is asymptotically non-zero, but the industry shocks are uncorrelated to this covariance term. They show that if the industry-level shocks are as-good-as-randomly assigned, conditional on $\mathbf{E}[\hat{\eta}_{jct-1}\Delta u_{ict}^{G}]$, the condition for instrument validity is still satisfied. In our framework, this condition would hold if $\hat{\eta}_{jct-1}$ predicted Δu_{ict}^{G}, but the industry wage shock, $\Delta\nu_{it}$, is uncorrelated with these predictions, so that $\sum_{j\in S}\Delta\nu_{jt}\mathbf{E}[\hat{\eta}_{jct-1}\Delta u_{ict}^{G}]$ is zero.

The sufficient conditions for IV consistency are therefore either an assumption that base-period non-manufacturing industrial composition does not predict future changes in comparative advantage or an assumption that the industry level shocks are as-good-as random with respect to $\mathbf{E}[\hat{\eta}_{jct-1}\Delta u_{ict}^{G}]$.

Our instruments can be written as:

$$\begin{aligned}
IV_{ct}^{W} &= \sum_{j\in S}\hat{\eta}_{jct-1}\Delta\hat{\nu}_{jt},\\
IV_{ct}^{B} &= \sum_{j\in S}\hat{\nu}_{jt}\Delta\hat{\eta}_{jct} = \sum_{j\in S}\hat{\eta}_{jct-1}\left[\frac{1+g_{jt}}{\sum_{j\in S}\hat{\eta}_{jct-1}(1+g_{jt})}-\frac{1}{\sum_{j\in S}\hat{\eta}_{jct-1}}\right]\nu_{jt}\\
&= \sum_{j\in S}\hat{\eta}_{jct-1}g_{jc}^{*}\nu_{jt}
\end{aligned}$$

where $g_{jc}^{*}=\left[\frac{1+g_{jt}}{\sum_{j\in S}\hat{\eta}_{jct-1}(1+g_{jt})}-\frac{1}{\sum_{j\in S}\hat{\eta}_{jct-1}}\right]$. As stated in the text, we are interested in the cross-city covariance between our instruments and the error term:

$$\frac{1}{I}\frac{1}{C}\sum_{c}\sum_{i\notin S}\left(\sum_{j\in S}\hat{\eta}_{jct-1}\Delta\hat{\nu}_{jt}\right)\Delta u_{ict}^{G}=\frac{1}{I}\frac{1}{C}\sum_{i\notin S}\sum_{j\in S}\Delta\hat{\nu}_{jt}\sum_{c}\hat{\eta}_{jct-1}\Delta u_{ict}^{G}$$

$$\frac{1}{I}\frac{1}{C}\sum_{c}\sum_{i\notin S}\left(\sum_{j\in S}\hat{\eta}_{jct-1}g_{jc}^{*}\hat{\nu}_{jt}\right)\Delta u_{ict}^{G}=\frac{1}{I}\frac{1}{C}\sum_{i\notin S}\sum_{j\in S}\hat{\nu}_{jt}\sum_{c}\hat{\eta}_{jct-1}g_{jc}^{*}\Delta u_{ict}^{G}$$

Thus, a sufficient condition for our instruments to be valid is that $\frac{1}{C}\sum_c \hat{\eta}_{jct-1}\Delta u^G_{ict} \to^p 0$ as $C \to \infty$ as stated in the text. Note that each instrument weights this condition differently; either $\Delta\hat{\nu}_{jc}$ or $\nu_{jc} \cdot g^*_{jc}$.

While we cannot test this assumption directly, we do attempt to assess its plausibility in several ways. Goldsmith-Pinkham et al. (2020); Borusyak et al. (2018) suggest checking whether observable baseline city-level characteristics are correlated with the instruments as an indirect exogeneity assessment. The idea is that if the Bartik-style instruments are correlated with baseline local characteristics that might be might be correlated to the structural error term in the estimating equation, then the consistency condition might not be met. In addition, Goldsmith-Pinkham et al. (2020); Borusyak et al. (2018) suggest controlling for base-period observable characteristics interacted with time trends when estimating 2SLS using the Bartik instruments. Borusyak et al. (2018), in particular, recommends an analysis at the industry level and controlling for industry level controls. Since our specification is at the city-industry level, we control for a full set of industry-by-year fixed effects, and specifically control for baseline or lagged manufacturing share.

Finally, we leverage the fact that we have two instruments and perform an over-identification test as done in Beaudry et al. (2012, 2018). As they discuss, each instrument uses a different type of variation, but are valid under the same identification assumption. In particular, each instrument can be seen as combining an industry-level shock with a local measure of exposure. Consistency depends on the orthogonality of the local measure of exposure (base-period non-manufacturing composition) and the error term. Given that each instrument weights potential violations of this assumption differently, if our orthogonality condition is not satisfied, estimates using either IV^W or IV^B should diverge. Using this insight, performing a standard over-identification test tests whether the instruments produce statistically different estimates. Note that this test is consistent with the theoretical framework. Each instrument should have the same impact on wages, because they both influence the outside options of workers in the same way regardless of whether the variation stems from shifts in industrial premia, IV^W, or because of shifts in industrial composition, IV^B. Likewise, what matters for firms is the bargained wage, so that each instrument should produce the same response on the firms' side. Thus, we expect that each instrument should produce similar estimates of the wage response in our structural equation. This is discussed in more detail below.

Heterogeneity Let Z_c represent an instrument, and let $\tilde{Z}_c$ represent the residual from regressing Z_c on industry dummies and city-employment rate (as in our main specification). Our 2SLS estimate of the wage coeffiicent in the gravity equation is given by (ignoring the t subscript for simplicity):

$$\begin{aligned}
\hat{\phi}_1^{Z_c} &= \frac{\widehat{\text{Cov}}\left(\tilde{Z}_c, \Delta \ln X_{icF}/R_{ic}\right)}{\widehat{\text{Cov}}\left(\tilde{Z}_c, \Delta \ln w_{ic}\right)} \\
&= \frac{\sum_c \sum_i \tilde{Z}_c \Delta \ln X_{icF}/R_{ic}}{\sum_c \sum_i \tilde{Z}_c \Delta \ln w_{ic}} \\
&= \frac{\sum_c \sum_i \tilde{Z}_c \phi_{1i} \Delta \ln w_{ic}}{\sum_c \sum_i \tilde{Z}_c \Delta \ln w_{ic}} + \frac{\sum_c \sum_i \tilde{Z}_c \Delta u_{ic}^G}{\sum_c \sum_i \tilde{Z}_c \Delta \ln w_{ict}} \\
&= \sum_i \phi_{i1} \cdot \frac{\sum_c \tilde{Z}_c \Delta \ln w_{ic}}{\sum_i \sum_c \tilde{Z}_c \Delta \ln w_{ic}} + \frac{\sum_i \sum_c \tilde{Z}_c \Delta u_{ic}^G}{\sum_i \sum_c \tilde{Z}_c \Delta \ln w_{ict}} \\
&= \sum_i \phi_{i1} \cdot \frac{\sum_c \tilde{Z}_c \frac{\hat{\gamma}_1}{\hat{\gamma}_2} Z_c}{\sum_i \sum_c \tilde{Z}_c \frac{\hat{\gamma}_1}{\hat{\gamma}_2} Z_c} + \frac{\sum_i \sum_c \tilde{Z}_c \Delta u_{ic}^G}{\sum_i \sum_c \tilde{Z}_c \Delta \ln w_{ict}} \\
&= \sum_i \phi_{i1} \cdot \frac{\frac{\hat{\gamma}_1}{\hat{\gamma}_2} \sum_c \tilde{Z}_c Z_c}{\frac{\hat{\gamma}_1}{\hat{\gamma}_2} \sum_i \sum_c \tilde{Z}_c Z_c} + \frac{\sum_i \sum_c \tilde{Z}_c \Delta u_{ic}^G}{\sum_i \sum_c \tilde{Z}_c \Delta \ln w_{ict}} \\
&= \sum_i \phi_{i1} \cdot \frac{1}{I} + \frac{\sum_i \sum_c \tilde{Z}_c \Delta u_{ic}^G}{\sum_i \sum_c \tilde{Z}_c \Delta \ln w_{ict}} \\
&= \phi_1 + \frac{\sum_i \sum_c \tilde{Z}_c \Delta u_{ic}^G}{\sum_i \sum_c \tilde{Z}_c \Delta \ln w_{ict}},
\end{aligned}$$

where $\sum_i \sum_c \tilde{Z}_c \Delta u_{ic}^G \to^p 0$ as $C \to \infty$ under the assumption that $\frac{1}{C} \sum_c \hat{\eta}_{jct-1} \Delta u_{ict}^G \to^p 0$ as $C \to \infty$. Note that $\phi_1 = \frac{1}{I} \sum_i \phi_{1i}$ is an average of potentially heterogeneous, industry specific effects. Thus, our Bartik-style or shift-share IV approach estimates a weighted average of unit specific treatment effects. As the coefficient on wages only varies across industries, our approach is analogous to estimating our gravity equation by industry and then averaging. Thus, what we estimate is an average of an industry specific effect. Since our regressions are weighted by the number lagged number of establishments in each city-industry cell, this is a weighted average.

Over-identification The over-identification test is:

$$\hat{\phi}_1^{IV^W} - \hat{\phi}_1^{IV^B} = \frac{\sum_i \sum_c \tilde{IV}_c^W \Delta u_{ic}^G}{\sum_i \sum_c \tilde{IV}_c^W \Delta \ln w_{ict}} - \frac{\sum_i \sum_c \tilde{IV}_c^B \Delta u_{ic}^G}{\sum_i \sum_c \tilde{IV}_c^B \Delta \ln w_{ict}} = 0$$

This condition will hold under any of the three conditions:

1. The exogeneity conditions for the instruments hold as stated in the text,
2. $\tilde{IV}_c^W = \tilde{IV}_c^B$ or are proportional. Since both instruments are based on the base-period industrial structure, this would occur if the national-level shocks used in each instrument were the same. This is easily rejected in our data: the correlation between our instruments is 0.32 after taking out year-effects, as we do in all of our estimations.
3. (1) and (2) don't hold, but the two terms just happen to balance. We view this as an unlikely 'knifes-edge' scenario.

G Data

This study uses two different data sources: the weakly anonymous Sample of Integrated Labour Market Biographies (SIAB) [Years 1975 - 2010] and the Linked-Employer-Employee Data (LIAB) [cross-sectional model 2 1993-2010 (LIAB QM2 9310)] from the Institute of Employment Research (IAB).

Data access was on-site at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the University of Michigan, the Cornell Institute for Social and Economic Research and subsequently via remote data access.[7]

SIAB Data. The SIAB data is a 2% random sample of individual accounts drawn from the Integrated Employment Biographies (IEB) data file assembled by the IAB. These data cover all employees registered by the German social insurance system and subject to social security. Civil servants and self-employed workers are not covered. The SIAB provide spell-data information on individual characteristics as such as gender, year of birth, nationality or education, and document a worker's entire employment history, e.g. an individual's employment status, full- or part-time status, occupational status, occupation and daily wage. Hours of work are not included in the IEB. Earnings exceeding the contribution assessment ceiling for social insurance are only reported up to this limit.[8] Administrative individual data are supplemented with workplace basic information taken from the Establishment History Panel (BHP). Establishment variables are measured on June 30 of each year and include information on location, industry, year of first and last appearance of the establishment, total number of employees, number of full employees, number of part-time employees and median wage of the establishment. Establishment and individual data are merged using employment spells which cover June 30.

LIAB Data. The LIAB data matches the IAB Establishment Panel data with individual social security data from the IAB on June 30 and comprises data from a representative annual establishment survey, stratified according to establishment size, industry and federal state. The survey provides information on establishment-level exports, employment and other performance-related measures, such as sales. For consistency with theory, we refer to these establishments as firms in the empirical analysis.

Cities and Industries. We define cities according to Kropp & Schwengler (2011) definition of labor markets. There are 24 cities; 19 in West Germany and 5 in East Germany.[9] There are 58 industries ("Abteilungen"), of which 29 belong to the manufacturing sector, grouped according to the 1993 time-consistent 3-digit classification of economic activities. In compliance with the FDZ guidelines, each industry-city cell includes at least 20 workers' observations.

Construction of the Main Variables. We use the LIAB data to construct industry-city-specific export shares in revenues and firm-level domestic revenues. We first compute firm-level export values using sales and the share of exports in sales, which are both available at the firm level in the LIAB data. Firm-level domestic revenues are obtained by substracting exports from sales. The industry-city export shares are obtained by aggregating firm revenues and exports by industry-city-year, weighting each observation using the weights provided in the establishment survey.

The SIAB data are used to construct industry-city wages, national industrial wage premia, predicted and observed local industrial employment shares, instruments, local employment rates, demographic controls and our proxy for local demand. These variables are then merged to the LIAB data by the Institute of Employment research.

Adjusted wages, predicted employment shares and instruments are constructed following the procedure described in Section 3.2. To estimate log industry-city wages from the wage regression at the worker level we first transform wages into real wages using the consumer price index, base 2005, provided by the German federal statistical office. Among the variables included in the vector of individual characteristics, our educational variable includes the following categories: without vocational training,

[7]See Heining et al. (2013), Fischer et al. (2009) and Heining et al. (2014) for further data documentation.

[8]We drop top coded observations.

[9]Kropp & Schwengler (2011) correspondence table between districts, labor markets and regions can be downloaded at http://www.iab.de/389/section.aspx/Publikation/k110222301.

apprenticeship, high school with *Abitur*, high school without *Abitur*, polytechnic, university. The nationality variable is restricted to two categories; German nationals and foreigners. In the second step which estimates the national industrial wage premia, we weigh observations by the size of the city-industry in the base-period so that the influence of each observation is proportional to its importance in that year.

Finally, we proxy changes in local demand, $\Delta \ln A_{ict}$, in the domestic revenue equation by interacting industry fixed effects with the traditional Bartik variable, constructed as a weighted sum of national-level industrial employment growth, where the weights are past local industrial shares.

H Additional Specification Checks

In this section, we investigate additional robustness exercises to probe the validity of our Bartik-type instruments by performing several specification checks suggested by Goldsmith-Pinkham et al. (2020). First, we assess the correlation between our instruments and characteristics of cities in the base year. In particular, we compute, by city, several variables aimed at capturing labor market conditions and city-skill in the base year. We then investigate the relationship between these variables and the base-period industrial structure. The idea is that if the instruments (through initial industry shares) are correlated with city characteristics in the base year, then any trend or shock that is correlated to those city characteristics could also be correlated with the instruments, therefore potentially violating the exclusion restriction we require for our instruments to be valid.

Table H.1 contains the results of this exercise. In the first two columns, we regress the value of our within- and between-instruments in 1996 (the first year we can calculate the instruments) on shares of college-educated, female, and German workers and the log employment rate and size of the workforce, average over the period 1992-1993. In these two columns, only one coefficient is statistically significant but the variables are jointly significant.

Since our identification strategy rests on the assumption that the initial industrial structure is not correlated with the residual in our second-stage regressions, we investigate the relationship between the initial industrial structure and city characteristics. In column 3, we compute the first principle component of our 58 industrial categories (i.e. the component that explains most of the variance in industry shares) in the base year. The idea is simply to reduce the dimension of our industrial categories into a single dimension that we can regress on our vector of city characteristics. Finally, in columns 4 and 5 we repeat these exercises by simply splitting industries into durables and non-durables. While the base-year characteristics are rarely individually significant, we cannot reject that they are jointly correlated with initial industrial structure.

Table H.1: Relationship between industry shares and city-specific characteristics

	IVB$_{ct}$ 1996	**IVW$_{ct}$ 1996**	**Comp. 1 1992-1993**	**Non-dur. 1992-1993**	**Durables 1992-1993**
City-specific characteristics in 1992-1993:					
Share of college graduates	0.005	0.008	-0.194	0.771	-0.771
	(0.003)	(0.008)	(0.134)	(1.034)	(1.034)
Share of females	-0.004	-0.017**	-0.044	-0.347	0.347
	(0.003)	(0.006)	(0.144)	(0.784)	(0.784)
Share of Germans	0.001	-0.005	-0.056	0.639	-0.639
	(0.002)	(0.005)	(0.146)	(0.643)	(0.643)
Log employment rate	0.005	-0.009	0.748***	-0.571	0.571
	(0.003)	(0.007)	(0.181)	(0.889)	(0.889)
Log workforce	-0.0001	0.0002	-0.081	0.027	-0.027
	(0.0002)	(0.0002)	(0.105)	(0.028)	(0.028)
Observations	24	24	24	24	24
R-squared	0.618	0.519	0.909	0.450	0.450
F-stat	5.82	3.88	35.94	2.94	2.94
p-val > F-stat	0.00	0.01	0.00	0.04	0.04
Proportion			0.27		

Notes: IVW$_{ct}$ corresponds to $\sum_i \hat{\eta}_{ict-1}\Delta\hat{\nu}_{it}$ and IVB$_{ct}$ to $\sum_i \hat{\nu}_{it}\Delta\hat{\eta}_{ict}$. The term 'Component 1' refers to the first principal component of industry shares in 1992-1993. In column 3, the first principal component and the city-specific characteristics are standardized to have unit standard deviation. The term 'Proportion' refers to the proportion of the variance of industry shares explained by the first principal component. The term 'Durables' ('Non-durables') refers to the share of employment in industries that produce durable (non-durable) goods in 1992-1993. Standard errors in parentheses.

I Additional Results

Table I.2: Trade Exposure and Other Outcomes

	Population			Wages		
	(1) EE	(2) China	(3) Both	(4) EE	(5) China	(6) Both
Δ Import Exposure	-0.017**	-0.014**	-0.0028	-0.00091	-0.0024	-0.000060
	(0.0052)	(0.0068)	(0.0021)	(0.0011)	(0.0025)	(0.00026)
Δ Export Exposure	0.0065**	0.0069	-0.00078	0.0012	-0.024**	0.00050
	(0.0032)	(0.0082)	(0.0012)	(0.00086)	(0.0071)	(0.00033)
Constant	0.14**	0.21**	0.18**	0.025**	0.20**	0.028**
	(0.046)	(0.049)	(0.047)	(0.0060)	(0.044)	(0.0049)
Observations	652	652	652	652	652	652
R^2	0.413	0.390	0.446	0.951	0.442	0.952
Predicted Impact:						
Mean	0.97	0.98	0.98	1.00	0.96	1.00
Med	0.98	0.98	0.98	1.00	0.97	1.00
10th pct.	0.95	0.95	0.96	1.00	0.93	1.00
90th pct.	0.99	0.99	0.99	1.01	0.98	1.01

Note: Standard errors, in parentheses, are clustered at the level of 50 larger labor markets areas. (***), (**), and (*) denote significance at the 1%, 5% and 10% level, respectively. The Table presents regression results of (29), estimated on different city outcome variables over 326 cities of West Germany. The dependent variables are population growth (columns 1-3) and the growth of wages (column 4-6) at the city level. Δ Import Exposure (IPW_{ct}) and Δ Export Exposure (EPW_{ct}) are observed decadal changes (1988-1998 and 1998-2008) in import and export exposure, respectively.Specifically, $IPW_{ct} = \sum_i \frac{E_{ict}}{E_{it}} \frac{\Delta M^{G \leftarrow East}_{i(t+10)}}{E_{ct}}$, where Δ denotes a decadal time difference, $\frac{E_{ict}}{E_{it}}$ is city c's share of industrial employment and E_{ct} is city c manufacturing employment. $\Delta M^{G \leftarrow East}_{i(t+10)}$ denotes the change in imports from the East between t and $t + 10$. Δ Export Exposure (EPW_{ct}) is computed similarly using exports. Each specification includes a set of region-time fixed effects and city-specific controls (the share of employment in tradable goods industries, the share of high-skilled, foreign and female workers, as well as the percentage of routine/intensive occupations). In each column, we instrument import exposure using $IVIPW_{ct} = \sum_i \frac{E_{ic(t-10)}}{E_{(it-10)}} \frac{\Delta M^{Others \leftarrow East}_{i(t+10)}}{E_{c(t-10)}}$, where $\Delta M^{Others \leftarrow East}_{i(t+10)}$ denotes changes in imports from the East to other high income countries. We instrument export exposure in a similar way using exports. We weigh our regressions by the share of the population in year 1978. In columns 1 and 4 (2 and 5), IPW_{ct} and EPW_{ct} are computed using imports from and exports to Easter Europe (China) only. In columns 3 and 6, IPW_{ct} and EPW_{ct} reflect trade exposure with both Eastern Europe and China.

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